

Original Research Paper

# The Cosmic Microwave Background: A Condition of Maximum

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Article history

Received: 04-12-2015

Revised: 05-02-2016

Accepted: 06-02-2016

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**Abstract:** It is shown that by imposing a condition of maximum value for the luminosity of the source at the origin of the cosmic microwave background, one arrives at a set of equations characteristic to important cosmological quantities. Although these equations do not have a solid theoretical support, the values of the parameters derived from them agree surprisingly well with the values given by the most recent measurements.

**Keywords:** Cosmic Background Radiation, Cosmological Parameters

## Introduction

After its discovery in (Penzias and Wilson, 1965) following several hints (Adams, 1941; McKellar, 1941) and predictions (Gamow, 1948; Alpher and Herman, 1948) the Cosmic Microwave Background (CMB) became one of the most studied phenomena in the modern cosmology. Its characteristics (black body spectrum, temperature, anisotropy, etc.) were measured with an increasing accuracy that culminated with the recent Planck 2013 collaboration (Planck Collaboration *et al.*, 2014). In this study we impose a condition of maximum value for the luminosity of the source at the origin of the CMB and using the familiar laws of physics arrive at a set of equations pertaining to prominent cosmological quantities. As an exercise, we derive from these equations three of the most important cosmological parameters and compare their values with those obtained from the most recent measurements.

## Materials and Methods

Let us consider a Comoving Volume (CV) of space of size  $R_{cv}$  at redshift  $z$ , corresponding to a radius  $R_0 = c/H_0$  at the present epoch. (Here  $c$  is the speed of light,  $H$  is the Hubble constant and the subscript "zero" denotes the present time). The radius  $R_{cv}$  is now equal to the radius  $R_{sls} = c/H$  of the Speed of Light Sphere (SLS), also known as the Hubble sphere (Ellis and Rothman, 1993), but because in decelerating universes the Hubble surface recedes faster than the galaxies and the universe was decelerating for most part of its life, in the past  $R_{cv}$  was much larger than  $R_{sls}$  and the SLS was inside our CV (Harrison, 1991). The matter in our CV (assumed for simplicity to be formed only of protons and electrons) is characterized by the baryon density  $\rho_b$ , the baryon

number density  $n_b = \rho_b/m_p$ , where  $m_p$  is the mass of the proton and the Thomson cross section of the main scattering particles  $\sigma_T = (8\pi/3) r_e^2$ , where  $r_e$  is the classical radius of the electron. The radiation is characterized by the temperature  $T_\gamma$  and the energy density  $u_\gamma = a T_\gamma^4$ , where  $a$  is the radiation density constant.

Taking into account that CMB was once by far the most dominating form of energy in the universe and the time derivative  $dE/dt$  of any amount of gravitational energy cannot exceed  $c^5/G$ , where  $G$  is the gravitational constant, the luminosity  $L_\gamma$  of a hypothetical source at its origin was most probably close to this value. It is therefore reasonable to assume that CMB luminosity corresponds to the gravitational collapse of a volume of space that released a quantity of energy equivalent to the maximum value in baryonic matter, in the minimum time allowed by causality. This does not necessarily mean that such an event actually occurred sometime in the past. It is simply a condition of maximum, as many of this type encountered in physics, imposed to our system in order to see how it affects its characteristics.

The energy released per unit volume in a gravitational collapse is limited from above by  $\rho_b c^2/2$ , while its time frame is limited from below by the expansion rate  $H$ . This constrains the volume of space that undergone transformation inside our CV to that of the SLS and makes the process similar to the collapse of the core of a massive star. The obvious choice for the maximum luminosity is therefore:

$$L_\gamma = \frac{2\pi}{3} R_{sls}^3 \rho_b c^2 H \quad (1)$$

With  $\rho_b = (3/8\pi) (\Omega_b H^2/G)$ , where  $\Omega_b$  is the baryon density parameter, one obtains:

$$L_\gamma = \frac{\Omega_b}{4} \frac{c^5}{G} \quad (2)$$

Note that for  $\Omega_b = \text{constant}$  this quantity does not depend on the size of the *SLS* (and hence the redshift) at the time when the hypothetical event occurred.

The radiation energy  $E$  inside a certain volume of space of radius  $r$ , due to an internal source of luminosity  $L$ , is given by  $E = L \sigma_T n_e r^2/c$ , where  $n_e = n_b$  is the electron number density. In the case of our *CV*, with  $n_b = \rho_b/m_p$ , one has:

$$E_\gamma = \frac{L_\gamma \sigma_T}{c} n_b R_{cv}^2 \quad (3)$$

From this equation, after dividing by  $(4\pi/3) R_{cv}^3$ , one obtains the following expression for the radiation energy density at a certain epoch:

$$aT_\gamma^4 = \frac{3}{4\pi} \frac{L_\gamma \sigma_T}{c} \frac{n_b}{R_{cv}} = \Omega_b \frac{r_e^2 c^4}{2G m_p} \frac{\rho_b}{R_{cv}} \quad (4)$$

Because the *CMB* temperature varies with the redshift as  $T_\gamma \propto 1+z$ , while the size and density of a *CV* vary as  $R_{cv} \propto (1+z)^{-1}$  and  $\rho_b \propto (1+z)^3$ , respectively, for  $\Omega_b = \text{constant}$  this equation is practically valid at any epoch.

Since a source of radiation exerts pressure on the surrounding matter, one can think of it as a “radiation charge”. Similarly to the case of assembling a sphere of uniformly distributed electric charges, one has to spend energy for assembling a spherical cloud of protons and electrons around a radiation source of luminosity  $L$ . The radiation force on a spherical shell of radius  $r$  and thickness  $dr$  is  $dF_\gamma = (L \sigma_T n_e/c) dr$ . The potential energy required to assemble a sphere of radius  $R_{cv}$  is, then:

$$W_\gamma = \int_0^{R_{cv}} r dF_\gamma = \frac{1}{2} \frac{L \sigma_T n_e R_{cv}^2}{c} \quad (5)$$

As expected, this is half the total energy in Equation 3 in accordance with the virial theorem. Therefore, one can define a radiation charge  $Q_\gamma$  as  $Q_\gamma^2 = E_\gamma R_{cv}$  and write:

$$Q_\gamma^2 = \frac{L_\gamma \sigma_T}{c} n_b R_{cv}^3 = \Omega_b G M_b \frac{r_e^2 c^4}{2G^2 m_p} \quad (6)$$

Where:

$$M_b = \frac{4\pi}{3} \rho_b R_0^3 = \frac{\Omega_b c^3}{2GH_0} \quad (7)$$

Is the baryonic mass inside our *CV*. Equation 4 then takes the simple form:

$$aT_\gamma^4 = \frac{3}{4\pi} \frac{Q_\gamma^2}{R_{cv}^4} \quad (8)$$

We note that the fraction at the right hand side of Equation 6 represents the mass of a fully ionized photon mean free path sphere (*PhMFPS*) formed only of protons and electron. This sphere is a hypothetical system characterized by a critical density and an optical depth equal to unity (Dinculescu, 2009). Its mass  $M_{ph}$ , radius  $R_{ph}$  and density  $\rho_{ph}$  follow from the constraints  $\sigma_T n_e R_{ph} = 1$  and  $\rho_{ph} = (3/8\pi) c^2 / G R_{ph}^2$ . One has:

$$M_{ph} = \frac{r_e^2 c^4}{2G^2 m_p} = \frac{R_{ph} c^2}{2G} \quad (9)$$

$$R_{ph} = \frac{r_e^2 c^2}{G m_p} \quad (10)$$

$$\rho_{ph} = \frac{3}{8\pi} \frac{G m_p^2}{r_e^4 c^2} \quad (11)$$

One can write therefore Equation 6 as  $Q_\gamma^2 = \Omega_b G M_b M_{ph}$ . For  $M_b = M_{ph}$  one has:

$$Q_\gamma^2 = \Omega_b G M_b^2 = \Omega_b \frac{r_e^4 c^8}{4G^3 m_p^2} \quad (12)$$

Here we would like to make a short parenthesis to present a point of view. Based on the Copernican Principle (Bondy, 1952) it was often argued that there should be nothing special about our *CV* because there is nothing special about our epoch. While fully agreeing with the fact that generally speaking there is nothing special about our epoch, we point out that with regard to the baryonic mass of our *CV* the present epoch is in some way special. It is the only epoch at which the strength of gravity of the baryonic mass is a factor of  $\Omega_b$  smaller than that of the *PhMFPS*. This characteristic might explain why apparently  $M_b = M_{ph}$ ,  $R_0 = R_{ph}/\Omega_b$  and  $\rho_b = \Omega_b^3 \rho_{ph}$ . (See the next section).

With  $N_b = M_{ph}/m_p$  and  $n_b = n_e$ , the total number of particles in our *CV* is:

$$N_{tot} = 2N_b = \left( \frac{r_e c^2}{G m_p} \right)^2 = \left( \frac{e^2}{G m_p m_e} \right)^2 \quad (13)$$

Where:

$e$  = The elementary electric charge in Gaussian units  
 $m_e$  = The mass of the electron.

We note in passing that this apparent relationship was suggested on rather obscure grounds more than 90 years ago by (Eddington, 1923).

From the last two equations one obtains the following expression for the radiation charge per baryon:

$$q_\gamma^2 = \Omega_b G m_p^2 \quad (14)$$

Knowing that the radiation pressure is one third of the radiation energy density, one arrives via Equation 8 and 12 at the following expression for the radiation pressure of the CMB:

$$P_\gamma = \Omega_b \frac{G M_b^2}{4\pi R_{cv}^4} \quad (15)$$

With  $n_b = \rho_b/m_p$ ,  $R_0 = R_{cv} (1+z) = R_{ph}/\Omega_b$ ,  $T_\gamma = T_\square (1+z)$  and the radiation entropy per unit volume in units of Boltzmann constant  $k$  given by:

$$s_\gamma = \frac{4}{3} \frac{u_\gamma}{k T_\gamma} \quad (16)$$

The dimensionless entropy per baryon is:

$$\frac{s_\gamma}{n_b} = \Omega_b^2 \frac{m_p c^2}{\frac{3}{2} k T_0} \quad (17)$$

Although perhaps of little significance, it is hard not to be surprised by the simplicity of the above equations and the familiar form of their numerical coefficients.

## Results

In order to substantiate our findings, let us try to derive from our equations three of the most important cosmological parameters and compare the obtained values at the present epoch with those given by the most recent measurements published by the Planck collaboration team. For these we are going to use the latest recommended values of the physical constants (Mohr *et al.*, 2012). We begin from the assumed equality  $M_b = M_{ph}$  and proceed as follows:

From  $2 G M_{ph}/R_{ph} c^2 = 1$  and  $2 G M_b/R_0 c^2 = \Omega_b$  one obtains  $R_0 = R_{ph}/\Omega_b$  and  $\rho_{b0} = \Omega_b^3 \rho_{ph}$ . In connection with Equation 10, this leads to:

$$R_0 = \frac{c}{H_0} = \frac{1}{\Omega_b} \frac{r_e^2 c^2}{G m_p} \quad (18)$$

Defining  $H_{100} = 100 \text{ km s}^{-1} \text{ Mpc}^{-1}$  and  $h = H_0 / H_{100}$  one can write:

$$H_0 = \left( \Omega_b h^2 \frac{G m_p H_{100}^2}{r_e^2 c} \right)^{1/3} \quad (19)$$

With  $\rho_{b0} = \Omega_b^3 \rho_{ph}$  and  $\rho_{ph}$  from Equation 11, one has:

$$\Omega_b = \left( \Omega_b h^2 \frac{r_e^4 c^2 H_{100}^2}{G^2 m_p^2} \right)^{1/3} \quad (20)$$

With one of the best known cosmological parameter  $\Omega_b h^2 = 0.02211$  (for definiteness) from (Planck Collaboration *et al.*, 2014), one obtains:

$$h = 0.6838 \text{ vs. } (0.661 - 0.685)$$

And:

$$\Omega_b = 0.04728 \text{ vs. } (0.0464 - 0.0511)$$

Knowing  $\Omega_b$  one can calculate  $Q_\gamma^2$  from Equation 12 and use it in Equation 8 to find the value of the redshift independent product  $R_0 T_0 = (3 Q_\gamma^2 / 4\pi a)^{1/4}$ . With  $R_0$  from Equation 18, this gives at once the present value of the CMB temperature:

$$T_0 = 2.72557 \text{ K vs. } (2.7249 - 2.7261) \text{ K}$$

## Discussion

The condition of maximum we imposed at the beginning of this paper led us to the following apparent characteristics of the CMB:

- The radiation charge of our CV is a factor of  $\sqrt{\Omega_b}$  smaller than the gravitational charge of its baryonic mass (Equation 12)
- The radiation pressure of the CMB inside our CV is a factor of  $\Omega_b$  smaller than the self-gravitational pressure of its baryonic mass (Equation 15)
- The entropy per baryon is a factor of  $\Omega_b^2$  smaller than the ratio of the rest energy of the proton to the kinetic energy of an electron in equilibrium with a thermal radiation with a temperature equal to the present CMB temperature (Equation 17)

One cannot refrain from asking oneself if all these apparent characteristics are mere coincidences or if they have a deeper meaning. Of course, one can brush aside all these relationships by invoking the anthropic principle (Barrow and Tipler, 1986), but this principle is too vague and difficult if not impossible to verify.

The equations we derived in this study are undoubtedly correct, as can be easily numerically verified, but are our conclusions sound? The history of science taught us that it is dangerous to draw conclusions based on numerical coincidences or on some apparent

relationships between physical quantities in the corresponding equations. Sometimes, however, this can be a path to the truth.

### Acknowledgement

I would like to express my gratitude to my life-long friend Professor Emeritus Alexandru T. Balaban for being such a good and open-minded mentor.

### Funding Information

The authors have no support or funding to report.

### Ethics

This article is original and contains unpublished material. The corresponding author confirms that all of the other authors have read and approved the manuscript and no ethical issues involved.

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