A Review on OFDM and PAPR Reduction Techniques

1Arun Kumar and 2Manisha Gupta

1Department of ECE, JECRC University, Jaipur, India
2Department of Physics, JECRC University, Jaipur, India

Abstract: OFDM is a multi carrier transmission technique where a single set of data is transmitted over a number of sub-carrier. OFDM takes the advantage of multi-path propagation and reduces the fading effect. The idea of OFDM is to split the total transmission bandwidth into a number of orthogonal subcarriers which reduces the inter-symbol-interference, power consumption and increases the capacity and efficiency of the system. Peak-Average-Power-Ratio (PAPR) is one of the disadvantages of OFDM which reduces the efficiency of system. In first part of this review, We have describe the OFDM characteristics of signals and its designing by combining the different block and in next part of this work, we have consider the PAPR effect in OFDM signals and have compared and review a several PAPR Reduction Techniques. The aim of this article is to provide a comprehensive detail about the implementation of an OFDM System and to study the different PAPR reduction techniques.

Keywords: OFDM, PAPR, FFT, IFFT, A Law, μ Law, Companding Technique, Clipping, Clipping and Filtering, TI, TR, Envelope Scaling, Wavelet

Introduction

In OFDM system a set of subcarrier are used that are orthogonal to each other in order to achieve a high spectral efficiency. The subcarriers in OFDM system will allow a subcarrier spectra to overlap with each other, hence increasing a spectrum efficiency. The signals are said to be orthogonal to each other if dot product of two deterministic signals is equals to zero (Louis and Michael, 2001). Basically an OFDM system is a multicarrier modulation system which divides the available spectrum into many subcarriers. Some of the applications of OFDM are WI-MAX, LTE Advance etc (Bastian et al., 2013). In a wireless broadcast system, OFDM plays an important role and reduce the complexity of receiver but in this process channel estimation and synchronization is very important (Vani and Reddy, 2013). In case of High speed communication, ISI is one of the major problems. The ISI is overcome on OFDM system by separating a guard-band to separate the channel (Urmila and Patel, 2014). The OFDM system is robustness to fading which is caused due to multi-path Interference. Cyclic Prefix is added to each symbol to overcome the ISI effect which leads to high spectral efficiency but it also cause ripples in power spectral density (Jiang et al., 2007). In OFDM system there is a fluctuation of amplitude in large amount which result in PAPR that cause non-linear distortion in practical implementation of amplifier which make the system inefficient (Han and Lee, 2005; Tao and Wu, 2008). Wavelet OFDM based Filter banks can be used to reduce the PAPR effect instead of using orthogonal subcarriers. Common phase error, phase noise also cause a subcarrier to lose their orthogonality by spreading the subcarrier to top of each other. The spreading of subcarrier is called Inter Carrier Interference (Souvik et al., 2013). One of the advantage of OFDM is that it achieve a sufficient flat channel due to a large number of subcarriers and it also achieve a same data rate of single carrier modulation at a lowest symbol rate. The symbols become longer due to the drop of symbol rate of each carrier.

Basic OFDM System Model

The OFDM modulated signal is given by following mathematical expression:

\[ Zn(t) = \sum_{n=0}^{N-1} S(n) e^{j2\pi f n t}, 0 \leq t \leq T_s \]
Where:

\[ T_s = \text{Symbol time} \]
\[ \Delta f = \text{spacing between subcarrier} \]
\[ N = \text{Number of Sub-channel} \]

The signal is orthogonal if it satisfies the condition \( T_s \Delta f = 1 \). In order to reduce the inter-symbol-interference cyclic prefix is added:

The transmitted signal with cyclic-prefix can be written as:

\[ T = T_g + T_m, \]

Therefore:

\[ Y_n(t + T_s) = \sum_{n=0}^{N-1} Y_n e^{j2\pi k_n f_0 t}, -T_s \leq t \leq T_s \]

The impulse response of a channel is given by the following equation:

\[ h(t) = h(t) = b_i \delta(t - d_i) \]

where, \( b_i \) and \( d_i \) are delay and complex amplitude.

The received signal is given by:

\[ X_n(t) = \sum Y_n x^k(t - ti) + d(t) \]

where, \( d(t) \) is noise of a signal.

**Characteristics of OFDM Signals**

Let us consider a block of \( N \) symbols \( Y = \{Y_k\} \), where \( k = (0, 1, 2, 3... , N-1) \) is formed with modulating symbols with set of subcarriers. The subcarriers are orthogonal to each other i.e., \( y_k^L = LAF \).

The ofdm symbols can be written as:

\[ y(t) = \frac{1}{N} \sum_{k=0}^{N-1} X_k e^{j2\pi k f t}, 0 \leq t \leq NT \]

where, \( j = \sqrt{-1} \). Now, Let us consider that input of OFDM signals are statistically independent and identically distributed if real part and imaginary part of OFDM signal are uncorrelated and orthogonal to each other. So considering the central limit theorem, where \( N \) is large, than the distribution of both real and imaginary signals approaches to Gaussian distribution with Zero and mean variance i.e.:

\[ a^2 = F [\Re(y(t))]^2 + \Im(y(t))^2 ] / \]

So the probability distribution function is:

\[ Pr(y(t)) = \frac{1}{\sqrt{2\pi\sigma}} e^{-y(t)^2/2\sigma^2} \]

The probability distribution function of OFDM signals when it is subjected to Rayleigh Channel is express as:

\[ Pr(r) = 2re^{-r^2} \]

where, \( r \) is the amplitude.

**OFDM System Design**

OFDM System design involves a lots of tradeoff's and differing necessities. The subsequent parameters could be a part of a general OFDM system requirement: (Bittner et al., 2008):

- Data Rate
- Available Bandwidth
- BER
- Delay spread of the channel

**Implementation of OFDM System**

The OFDM system is implemented by combining the different block as shown in Fig. 1.

An OFDM is a Multicarrier Modulation Technique that use a overlap signals to divide the frequency selective channel into a number of narrow band flat fading channel. The FFT encodes the block of symbol; instead of sending the data sequentially on a single carrier at a high symbol rate. The sub-channels are made orthogonal by spacing the subcarrier at the increase of symbol time. The multipath fading can be nullified by making the symbol period of sub-channel longer in their length as compare to multipath delay spread. The signals having high noise and interference is deactivated, thus decreasing the effect of fading and interference. OFDM modulation technique is generated through the use of complex signal processing approaches such as Fast Fourier Transforms (FFTs) and inverse FFTs in the transmitter and receiver sections of the radio. One of the benefits of OFDM is its strength in fighting the adverse effects of multipath propagation with respect to inter-symbol interference in a channel. OFDM is also spectrally efficient because the channels are overlapped and contiguous. The block diagram of OFDM is shown in above figure. In this system input data are FEC coded with technique such as convolution code. The diversity gain is obtained by interleaving the coded bit stream. The constellation points are map after a group of channel bits are grouped together. Now the data is serial which is represented by complex numbers. At this point Mapping Technique such as pilot mapped is used. A serial to parallel converter is used and IFFT is applied on the complex parallel data.
As per the need of transmission sub-carriers, the transformed data are grouped. In every block of data cyclic prefix is inserted and the data is multiplexed in serial fashion. Now the OFDM data are modulated and the digital data is converted into analog by using a DAC and RF modulation is also performed. The transmitted OFDM signals goes through all anomalies and hostility of wireless channels. The receiver perform the down conversion of the signal and convert the signal into digital domain by using ADC. The synchronization is needed during the down conversion of the signal. The OFDM signal is demodulated by using a FFT. The channel estimation is performed and complex receive data are de-mapped according to the constellation diagram. At last the original signal is received by using the FEC coding and Decoding (Syrjala and Valkama, 2010).

**Block of OFDM System**

**Random Data Generator**

The function of random generator is to generate a random uniform data in the range of (0, M-1), where M is the Mary number. The Mary can be either a scalar or a vector. If we define it as a scalar, then all output random variables are independent and identically distributed. If we consider the input signals as vector then the size of initial seed must be equal to size of Vector. If the initial seed is constant then the resulting noise is repeatable.

**Data Mapping**

A serial to parallel conversion of data is done according to the modulating technique used. The data is transmitted by assigning a unique word to each carrier. The symbols are allocated to each of the subcarriers which are phase mapped and are represented by a complex in-phase or quadrature in-phase (Namisha and Mohit, 2013).

**Serial to Parallel Converter**

The main function of serial to parallel converter is to convert the serial data parallely. The parallel data is transmitted by assigning a unique word to each of the subcarriers. Once the symbol has been allocated to each of the subcarriers then they are phased mapped according with modulating scheme.

**Digital Modulation Scheme Use in OFDM Transmitter**

It is one of the advantage of OFDM that different modulation scheme can be applied to each sub channel depends on channel condition, data rate, robustness, throughput and channel bandwidth. There could be different modulation scheme applied specified by complex number i.e., QPSK, 16 QAM. Modulation to OFDM sub channel can be made adaptive after getting information and estimation of channel at transmitter.

**IFFT-Frequency-Domain to Time Domain Conversion**

The orthogonality of subcarrier is maintained and the frequency domain signals are converted into a time domain and the generation of real-output signal is achieved by arranging the conjugate of sub-carrier. In this stage the techniques like IFFT Mapping, Zero mapping and selector bank is included to overcome the problem of length of subcarrier and bin size.

**Guard Period**

The last part of the OFDM signal is copied and inserted at the beginning of the signal. The length of
the signal should be equal or greater than the time dispersion of the signals. In this way increase the length of the Signal from $T_{Tu}$ + $T_{Cp}$. Again at the receiver the guard band is removed (Nicola et al., 2014). One of the proposals suggests using quiet bursts for spectrum sensing. During the quiet burst, some OFDM symbols in the sub-frames can be replaced by the sensing period. Another suggestion from the same source utilizes the proposed downlink-uplink switching time for spectrum sensing. This sensing interval is however asynchronous and the length of the sensing interval varies with distance from WRAN base-station. Some scheme was proposed to use the guard interval of OFDM symbol for spectrum sensing. This scheme is generic and does not depend on distance from the base-station. This scheme can detect primary users much quicker due to very high frequency of spectrum sensing. Most importantly, explicit sensing interval need not be allocated and thus it does not affect spectrum utilization (Quoc-Tuong et al., 2009).

**Guard Interval Insertion and Removal**

The sensitivity to the time synchronization is achieved and also it eliminates the pulse shaping filter. When the length of the guard interval is longer than the duration of the channel impulse response, ISI can completely be removed. However, as the transmission efficiency reduces with the insertion of the guard interval (during the guard interval, no new information can be transmitted), the guard interval must be chosen sufficiently small. The most commonly used guard interval is the Cyclic Prefix (CP).

**Parallel to Serial Conversion**

The final stage in the implementation must undo the first stage. A switch is used to time-division multiplex the four individual bit signals into a single sequence.

**AWGN Channel**

Additive white Gaussian Noise (AWGN) is a basic noise model used in Information theory to imitate the effect of many random processes that occur in nature.

**FFT: Time Domain to Frequency Domain Conversion**

OFDM distributes the data over a large number of carriers at different frequencies. This spacing provides the orthogonality which prevents the receivers to see wrong frequencies. In opposite to other multi-carriers techniques, like CDMA, OFDM prevents the Inter Symbol Interference (ISI) by adding a cyclic prefix one of the key features of OFDM is the IFFT/FFT pair. These two mathematical tools are used to transform several signals on different carriers from the frequency-domain to the time-domain in the IFFT (or FFT-1) and from the time-domain to the frequency-domain in the FFT.

**PAPR**

PAPR may be define as square of peak amplitude to the ratio of square of peak rms value. Mathematically it is defined as:

$$PAPR = \frac{I^2_{peak}}{I^2_{rms}}$$

It is one of the problem in OFDM system. The input symbol of IFFT posses a uniform power spectrum but the output of IFFT may result a non uniform or spiky power spectrum (Gangwar and Bhardwaj, 2012).

**PAPR Reduction Techniques**

Some of the method proposed to overcome the effect of PAPR and the different PAPR reduction technique is used accordance to the need of system (Urban and Marsalek, 2007; Duanmu and Chen, 2014; Liang, 2014; Bauml et al., 1996; Fischer, 2007; Biserka et al., 2010).

**Companding Technique**

In this technique, at the transmitter side the signal is compressed and again at the receiver the signal is decompressed. This techniques are used in lower resolution for higher amplitude and higher resolution for lower amplitude. A memory-less transformation is used to squeeze the signal at the transmitter side and by considering the approximate Rayleigh distribution of OFDM signals, the amplitude level are enlarged at the receiver side. The probability distribution of amplitude of OFDM signal is changed and reduction in PAPR is achieved by both compressing and decompressing the amplitude (Zolghadrasli and Ghamat, 2008; Nee and Prasad, 2000).

**µ-Law and A-Law Companding Technique**

This technique is used in digital telephone systems of USA and Japan. Mathematically, this standard is defined as:

$$F(x) = \text{sgn}(x) \frac{\ln(1 + \mu |x|)}{\ln(1 + \mu)} 1 <= x <= 1$$

where, $\mu$ is parameter of compression ($\mu = 255$ for USA and Japan) and $x$ is normalized value of the input signal. The Compression in this standard is defined as:

$$F(x) = \text{sgn}(x) \frac{(I + \ln A |x|)}{I + \ln A} / A <= |x| <= 1$$
where, $A$ is the compression parameter ($A = 87.6$ in Europe) and $x$ is the normalized value of the input signal.

**Clipping and Filtering**

The PAPR can be reduced by clipping the higher amplitude peaks. Several techniques of clipping are available. Some of the techniques clip the signal at the output of the inverse Fourier discrete transfer and some other techniques clip the signal after the interpolation and also used filter to reduce output band power. The techniques results in Spectral splattering and also introduce the additional noise to the signals (O'Neill and Lopes, 1995; Ojima and Hattori, 2009). This technique also efficiently removed the expanded spectrum. The filtering technique can reduce the spectrum growth but filtering after clipping can also reduce the out-band radiation. One of the disadvantage is that it may increase the peak level where peak signals exceed the clip levels (Matsuda et al., 2007). The iterative clipping and filtering reduce the PAPR effect without the spectrum expansion. This technique usually takes a long time also it makes the system more complex by increasing the complexity of an OFDM Transmitter (Nandalal and Sophia, 2014).

**Clipping**

This technique cancels the amplitude of the signal that exceeds the threshold value of amplitude. Thus decreasing the PAPR in the system. It also add a clipping noise which is due to the distortion of power and expand the signal spectrum of transmitter which cause the interference in the signal. Clipping is non linear techniques that produce in-band noise distortion which reduce the performance and efficiency of BER and out-band noise which reduce the overall spectrum efficiency (Snehal and Rathkanthiwar, 2013).

**Block Coding Scheme with Error Correction**

This method is proposed so that designed block codes not only minimize the PAPR, but also give error correction capability. A k bit data block is encoded by a (n, k) block code with a generator matrix “G” in the transmitter of the system. Followed by the phase rotator vector b to produce the encoded output $x = a.G + b$ (mod 2). After that generator matrix G and the phase rotator vector b are produced; which are used mapping between these symbols combination and input data vector a. The converse functions of the transmitter are executed in the receiver system. The parity check matrix H is achieved from the generator matrix G, with an exception that the effect of the phase rotator vector b is removed before calculations of syndromes (Chauhan et al., 2012).

**Interleaving Technique**

The notion that highly correlated data structures have large PAPR can be reduced, if long correlation pattern is broken down. The basic idea in adaptive interleaving is to set up an initial terminating threshold. PAPR value goes below the threshold rather than seeking each interleaved sequences. The minimal threshold will compel the Adaptive interleaving (AL) to look for all the interleaved, sequences. The main important of the scheme is that it is less complex than the PTS technique but obtains comparable result. This method does not give the assurance result for PAPR reduction (Muller and Huber, 1997).

**Tone Reservation (TR)**

The main idea of this method is to keep a small set of tones for PAPR reduction. This can be originated as a convex problem and this problem can be solved accurately. Tone reservation method is based on adding a data block and time domain signal. A data block is dependent time domain signal to the original multicarrier signal to minimize the high peak. This time domain signal can be calculated simply at the transmitter of system and stripped off at the receiver. The amount of PAPR reduction depends on some factors such as number of reserved tones, location of the reserved tones, amount of complexity and allowed power on reserved tones. This method explains an additive scheme for minimizing PAPR in the multicarrier communication system. It shows that reserving a small fraction of tones leads to large minimization in PAPR ever using with simple algorithm at the transmitter of the system without any additional complexity at the receiver end. Here, N is the small number of tones, reserving tones for PAPR reduction may present a non–negligible fraction of the available bandwidth and resulting in a reduction in data rate. The advantage of TR method is that it is less complex, no side information and also no additional operation is required at the receiver of the system (Baum et al., 1996).

**Tone Injection (TI)**

This technique can achieve a no lose in data after the reduction of PAPR multicarrier signals. The main motive is to increase the size of the constellation so that signal basic constellation can map into several equivalent point in the new extended constellation (Cimini and Sollenberger, 2000; Suma et al., 2014).

**Envelope Scaling**

The Envelope Scaling technique has been proposed by Foomooljareon and Fernando in (Negash and Nikookar, 2001). They introduced an algorithm to reduce PAPR by scaling the input envelope for some subcarriers before they are sent to IFFT. They used 256 subcarriers with QPSK modulation technique to achieve a equal envelopes of subcarriers. In this technique, the input envelope in some sub carrier is scaled in order to achieve the least amount of PAPR at the output of the IFFT.
Table 1. Comparison of PAPR reduction schemes

<table>
<thead>
<tr>
<th>PAPR reduction techniques</th>
<th>Decrease distortion</th>
<th>Power raise</th>
<th>Defeat data rate</th>
<th>Operation required at transmitter and Receiver</th>
</tr>
</thead>
<tbody>
<tr>
<td>Block coding</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>TX: Coding or table searching RX: Decoding or table searching</td>
</tr>
<tr>
<td>PTS</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>TX: V times idfts operation RX: Side information extraction, inverse pts</td>
</tr>
<tr>
<td>Clipping and filtering</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>TX: Clipping RX: No clipping</td>
</tr>
<tr>
<td>Interleaving</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>TX: D times idfts operation, D1 times interleaving RX: Side information extraction, de-interleaving</td>
</tr>
<tr>
<td>TR</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td></td>
</tr>
<tr>
<td>TI</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td></td>
</tr>
</tbody>
</table>

Thus, the receiver doesn’t need any feedback for decoding the receiver sequence. This scheme is appropriate for QPSK modulation.

**Wavelet**

The advantage of Wavelet analysis over Fourier analysis, as it allows a time-frequency representation domain, allowing optimal resolution and flexibility. Although the concept OFDM principle has been introduced a long time ago, but its realization is quite new and still has some challenges such as frame synchronization. Still, the classical implementation uses Fourier analysis. The techniques like wavelet and wavelet packet analysis have also been introduced for this modulation technique (Daneshgaran et al., 2009; Lindsey and Dill, 1995; Lindsey, 1997; Latif and Gohar, 2010). Wavelets analysis have some crucial advantages, but also it have some dis-advantage such as a greater sensitivity to imperfect sampling over the traditional analysis (Mahmudul and Singh, 2012; Miller et al., 2001; Okamoto et al., 2003; Jamin and Mahonen, 2005; Damavandi et al., 2013). Wavelet analysis can provide a better time-frequency localization which can also result in improved flexibility and use of signal spectrum. Whether it is a Fourier or wavelet method, still it poses a lot of challenges in OFDM systems. One of challenge is frame synchronization, Synchronization issues are well described in (Bianchi and Argenti, 2003; Luise et al., 2000) and intensive research work has been carried out (Jacklin and Ding, 2013; Azurdia-Meza et al., 2012), however all studies consider Fourier OFDM and those considering wavelets use Fourier OFDM classical analysis. The synchronization issues are generally surmount using either the Maximum Likelihood (ML) function approach or by leveraging learning sequences (Coulson, 2001). However in both cases, some performs bottlenecks.

**Conclusion**

OFDM is similar to Frequency division multiplexing but the difference lies in modulation and demodulation of signals. The basic priority is to minimization of Inter-symbol Interference, cross-Talk and PAPR Reduction. The different PAPR reduction schemes are applied accordance with the technology. OFDM has several interesting properties that suit its use over Wireless channels and hence many Wireless standards have started to use OFDM for modulation and multiple accesses.

It is also concluded that OFDM when combines with MIMO plays a very important role in next generation mobile communication system because of high demand in data speed and to compensate the growing capacity in limited spectrum. One of the drawback of MIMO-OFDM is its hardware implementation is very complex and its circuit require a high power which make it costly and its size is also large as compare to traditional antennas. This article provides the comprehensive details about the OFDM System and different PAPR reduction techniques which also gives an opportunity to the researcher and academician who wants to work in this field. OFDM is similar to FDM but the difference lies in modulation and demodulation of a signals. The basis priority is to minimization of ISI, Cross-talk and PAPR. The different PAPR schemes are applied accordance with the technology. The comparison of different PAPR schemes is given in Table 1.

**Author’s Contributions**

Arun Kumar: Have review, wrote all the paper and contributed to the writing of manuscript.

Manisha Gupta: Have guided and design the research plan.

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