

STOCHASTIC PROGNOSTIC PARADIGM FOR PETROCHEMICAL PIPELINES SUBJECT TO FATIGUE

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ABSTRACT

The most important aim for industrialists is the prevention and the evaluation of their products state since non-predicted failure is very expensive in some cases. This can be done mainly by the evaluation of the "Remaining Useful Lifetime" (RUL) by the means of prognostic approaches compensating the inconveniences of classical maintenance strategies. A proposed analytic prognostic methodology based on damage laws, such as Paris-Erdogan's and Palmgren-Miner's laws, is developed here to determine the RUL of the system. It permits to ensure a high availability and productivity with less cost for industrial systems. To make this approach more reliable, it is essential to introduce the stochastic description. For the case of fatigue effect where damage state is growing from macro-cracks to total failure, $D(N)$ expresses an increasing scalar damage function in terms of loading cycles N . The RUL is estimated from a predefined threshold of damage D_c . Pipelines tubes, subject to fatigue effects due to pressure-depression alternation, belong to vital mechanical systems in petrochemical industries that serve to transport natural gases or liquids. The prognostic evaluation of their states increases the tubes availability while minimizing their missions cost.

Keywords: Prognostic, Analytic, Damage, Stochastic, Fatigue, Pipelines

1. INTRODUCTION

The last phase during each system life represents the progressive degradation period. It is important to predict, at each instant, the remaining lifetime in order to prevent expensive breakdown and to avoid catastrophic failures. Adopting preventive systematic maintenance by frequent replacement to increase the system availability is an expensive strategy (Inman *et al.*, 2005).

The prognostic is a methodology aiming at predicting the RUL of a system in service (Vachtsevanos *et al.*, 2006). It can be expressed in hours of functioning, in Kilometers run, or in cycles. The prognostic is defined as the ability to "predict and prevent" possible fault or system degradation before failures occurrence (Lee, 2004). Maintenance actions can be taken ahead of time if the condition of machines and systems can be effectively predicted.

Several prognostic studies are proposed and are based on abaci of degradation without any analytic form (Peysson, 2009). A prognostic methodology based on analytic laws in fracture mechanics, such as the crack propagation and the damage accumulation laws, is proposed later (Abou Jaoude *et al.*, 2010). Whenever such analytic laws are available, this methodology seems to be interesting. A degradation indicator D was taken to describe the evolution from an initial micro damage till the total system failure.

In addition, by including a stochastic analysis on the previous model (Abou Jaoude *et al.*, 2010), it becomes more precise for RUL prediction since it covers more possible realizations. This is done by considering some parameters as random variables.

Our aim is to prepare a general prognostic tool that can be capable of well predicting the RUL of a system

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based on an analytical damage accumulation in either deterministic or stochastic context. Knowing that, the fatigue RUL can be expressed in terms of: crack length a_C , or critical number of loading cycles N_C , or material tenacity K_{IC} , from which various limit states can be written.

The life prognostic of petrochemical pipelines is vital in their domain since their availability has crucial consequences. Fatigue failure is their main failure cause due to internal pressure-depression variation along the time. Usually, three situations for these pipes exist: unburied, buried and under sea water (offshore pipes). Each one of these situations requires different physical parameters like: Corrosion, soil pressure and friction, water and atmospheric pressure.

In the present work, the prognostic study is applied to unburied pipes taking into account an analytical damage accumulation and considering two random variables which are the internal pressure P_0 with Triangular distribution and the initial crack length a_0 with Log-Normal distribution.

The study is organized as follows: first a detailed state of the art is elaborated, second a stochastic prognostic paradigm is proposed, third an application to petrochemical pipelines is considered, fourth the results of corresponding simulations are shown, fifth a flowchart summarizes the methodology and finally, a conclusion is presented.

1.1. State of the Art: Stochastic Fatigue Modeling

The science and technology of prognosis and structural health management offer the potential for significant enhancements in the safety, reliability and availability of high-value resources (Christodoulou and Larsen, 2004). This concept is based on a closed-loop process whose successful implementation depends on the integration of several multi-disciplinary elements including (Hudak *et al.*, 2002):

- Onboard sensing of operational parameters and material damage states
- Diagnosing trends, fault conditions and underlying damage
- Predicting remaining useful life in terms of probability of failure and limits on reliable performance
- Deciding upon appropriate courses of action: whenever or not the resource is capable of performing a given mission, or alternatively, is in need of inspection, maintenance, or replacement

Considerable uncertainty exists in the usage and sensor inputs, as well as the required modeling and

associated material property inputs. Consequently, there is an inherent need for the reasoning element of the prognosis system to be probabilistically-based.

In contrast, probabilistic life prediction is typically based on material property data, finite element thermal and stress analysis, pre-service inspection and in-service monitoring for defects and damage accumulation algorithms. The advantage of this approach is that it is more amenable to linkage with the underlying physical mechanisms of damage (i.e., crack nucleation and growth). Thus, the process is inherently suitable for extension into materials prognosis, a novel concept that seeks to combine information on the material damage state with mechanistically-based predictive models.

Probabilistic analyses of prognostic uncertainty were performed using a probabilistic life prediction code DARWIN (Leverant *et al.*, 1977) as a demonstration platform. DARWIN integrates finite-element stress analysis results, fracture-mechanics-based life assessment for low-cycle fatigue, material anomaly data, probability of anomaly detection and inspection/monitoring schedules to determine the probability-of-fracture of rotor disks as a function of operating cycles. In the study on lives of turbine engines (Hudak *et al.*, 2002), enhancements were added to the DARWIN code to enable the type of analyses required for prognosis: (1) Establishment of interface with engine sensor data; (2) Adding of the fatigue crack initiation analysis to existing fatigue crack propagation analysis; (3) Incorporates the integration of crack initiation and propagation algorithms including correlation effects between the two damage processes; (4) Adding a damage-based load filtering method to reduce computational time; (5) Capability to analyze a large number of inspections (or interrogation-up to once per flight cycle) to simulate continuous monitoring with an on-board sensor. Although DARWIN contains several probabilistic solutions methods, the analyses in reference to (Hudak *et al.*, 2002) were performed using Monte Carlo simulation.

Other models have been proposed to describe the random behavior of fatigue crack growth in metals. In Yang and Manning (2003) stochastic model a simple second order approximation of a deterministic crack growth model is used with a random component.

For instance, with transitional loading the model parameters will vary as the fatigue damage propagates. The model parameter variability was taken into account in the data driven part of the analytical crack exceedance probability, which is the probability that the crack length will exceed a number of cycles, with the respective load

period. To directly account for the variance in the crack growth rate, the random component is assumed to follow a Log-Normal distribution (Moreno *et al.*, 2003).

A significant part of main pipelines are subjected to external cracking, which is a serious problem for the pipeline industry like in Russia, U.S. and Canada and others (Sergeeva and Bolotov, 1996; Jaske, 2000). Identification of external cracks is achieved using different Nondestructive Evaluation (NDE) methods. If cracks are revealed during inspection, their influence on the Remaining Useful Lifetime (RUL) of the pipeline should be assessed in order to choose what maintenance action should be used: do nothing/repair/replace.

Pipeline integrity is assessed on the assumption that some defects after In-Line Inspection (ILI) may be: still undetected; detected, but not measured; detected and measured. It is possible to update the stochastic remnant life of pipelines using the data available due to ILI.

The works of Timashev *et al.* (2005) describe a new practical method of updating the stochastic remaining life of pipelines with defects using the latest ILI data. It describes a comprehensive algorithm for assessing pipeline remnant life taking into account the results of holistic statistical analysis of In-Line Inspection (ILI) data. It is assumed that the pipeline segment wall has a longitudinal external crack of semi-elliptical form and is described by the J-integral. The Limit State Function (LSF) is described as the difference of the critical and current value of the J-integral. The critical crack depth is defined using the notion of fracture toughness and the J-integral approach.

Model-based prognostic techniques rely on a dynamic model of the predicted process. This approach uses a mathematical model of the process in order to implement the physical understanding of the system into the diagnostic problem. Such models should describe both nominal and faulty behavior of the system. As a result, it is possible to explain the fault progress in time and to make End of Life (EOL) and RUL predictions. These methods involve the estimation of residuals as a deviation between the real system measurements and proposed model outputs. In the ideal case, the residuals are zero but in reality there are permanent noise and modeling errors. It is, therefore, expected that the residuals are small in the nominal working mode and larger in the presence of a failure. Once the residuals are obtained, it is possible to use some statistic representation to estimate the distribution of RUL as a function of present uncertainties and to calculate possible damage.

In this study, system modeling is considered by the physics-based prognosis derived by using physics laws and principles. Crack initiation based models must include all the available information about component and its environment (temperature, humidity, soil effects). The crack propagation models can be divided into two main groups: deterministic and stochastic. Deterministic crack propagation models, which usually describe the growth of the crack, are based on Paris (1963) law. Stochastic crack propagation involves models with random parameters which can be estimated using Monte Carlo simulations.

1.2. Stochastic Damage Accumulation

An analytical prognostic model recently developed (Abou Jaoude *et al.*, 2011a; 2011b) aims at giving a useful RUL prediction tool. This model uses the physical law of Paris-Erdogan for crack propagation (Paris, 1963) and the law of Palmgren-Miner for linear damage accumulation (Miner, 1945), to estimate the residual lifetime in fatigue failure risk. It consists of the evaluation of a normalized degradation indicator D ($0 \leq D \leq 1$) in terms of load cycle numbers N.

The fatigue failure is reached when the crack size "a" grows to a critical size a_c with respect to Paris' law where the necessary number of cycles is the critical number N_c . Using Miner's cumulative damage, after each load cycle, the damage indicator D increases by a relative crack length increment da as indicated by the following expression Equation1 (Abou Jaoude *et al.*, 2010):

$$D_i = \frac{1}{a_c - a_0} \sum_{j=1}^i da_j = \frac{a_i}{a_c - a_0} \quad (1)$$

The deterministic Paris' law is given by Equation 2:

$$\frac{da}{dN} = C \cdot [\Delta K(a)]^m; \quad (2)$$

$$\Delta K(a) = Y(a) \cdot \Delta \sigma \cdot \sqrt{\pi \cdot a}$$

Where:

- a_0 = The initial crack length
- a = The actual crack length
- N = The load cycle
- C and m = The material and environment parameters ($0 < C \ll 1$); ($2 \leq m \leq 4$) (Lemaitre and Chaboche, 1994)
- $\Delta K(a)$ = The stress intensity factor range
- $Y(a)$ = The geometric factor function of the body dimensions
- $\Delta \sigma$ = The applied stress range



Fig. 1. Summary of the ISO 13381-1: 2004 standard main steps

1.3. Stochastic Modeling

The stochastic modeling aims at considering some influent parameters as random variables and hence, the Paris' law becomes stochastic crack propagation law. From data provided from previous diagnostic analysis (Fig. 1), it is possible to consider the initial crack length a_0 as the main random variable where the second variable is the stress loading σ . Many other parameters can be also considered random and the prognostic model can be expressed by the following general function:

$$D(a) = P_{rog}(a) = fct(a_0, \sigma, \text{thickness } e, \text{ dimensions, } C, m, \dots)$$

The degradation indicator D variant from 0 to 1 provides instantaneously the Remaining Useful Lifetime (RUL) expressed by time, or cycle, or distance, depending on the type of device concerned. A probabilization of basic parameters leads to a probabilistic trajectory $\bar{D}(a)$.

All previously mentioned parameters are affected by some probability of realization that influences the resulting RUL deduced from $D(a)$. Contrary to the deterministic-based prognosis, the RULs concluded in stochastic-based prognosis are related to probabilistic aspect. As the estimated RUL is no longer deterministic, it is affected by some risk percentage in order to be realized.

These relevant basic parameters must be modeled stochastically using a convenient well known probability distribution laws. For example, the initial crack length a_0 is modeled by either a Normal or Log-Normal distribution, the loading σ is modeled by a Normal distribution.

1.4. Stochastic Basic Parameters

Here a Probability Density Function (PDF) of initial crack length a_0 that follows a Log-Normal distribution is considered, then Equation 3:

$$f_0(a_0) = \frac{1}{a_0 \xi \sqrt{2 \cdot \pi}} \cdot \exp\left[-\frac{1}{2 \cdot \xi^2} (\ln(a_0) - \lambda)^2\right] \quad (3)$$

With:

ξ = The standard deviation of the variable $\ln(a_0)$ which is the equivalent Normal distribution

λ = The mean of the variable $\ln(a_0)$

Expectation of a_0 :

$$E(a_0) = \exp[\lambda + \xi^2 / 2]$$

Variance of a_0 :

$$V(a_0) = \exp[2\lambda + \xi^2] \times (\exp[\xi^2] - 1)$$

Inversely, one can also write:

$$\lambda = \ln[E(a_0)] - \frac{1}{2} \ln\left(1 + \frac{V(a_0)}{E(a_0)^2}\right)$$

$$\xi^2 = \ln\left(1 + \frac{V(a_0)}{E(a_0)^2}\right) \Rightarrow \xi = \sqrt{\ln\left(1 + \frac{V(a_0)}{E(a_0)^2}\right)}$$

The limit value of crack length (a_C) is fixed when the number of cycles reaches the critical value (N_C) starting from an initial state N_0 (Fig. 2). The probability of fatigue failure is given by:

$$P_{rob}(a_N > a_C) = \int_{a_C}^{\infty} f_N(a_N) da_N$$

where, $f_N(a_N)$ is the PDF of the crack width $a(N)$ at cycle N .

It can be assumed that $a_C = e/8$ (Luo *et al.*, 2003), where e and ℓ are respectively the device dimension in the crack direction and the perpendicular dimension to the crack direction (Fig. 3). Δa_N is the crack length increment due to a loading cycle dN .

1.5. PDF of Crack Length a_N at Loading Cycle N

From Paris' law we can deduce Equation 4:

$$\frac{da}{dN} = C \cdot [\Delta K(a)]^m \Rightarrow \frac{da}{[\Delta K(a)]^m} = C \cdot dN \quad (4)$$

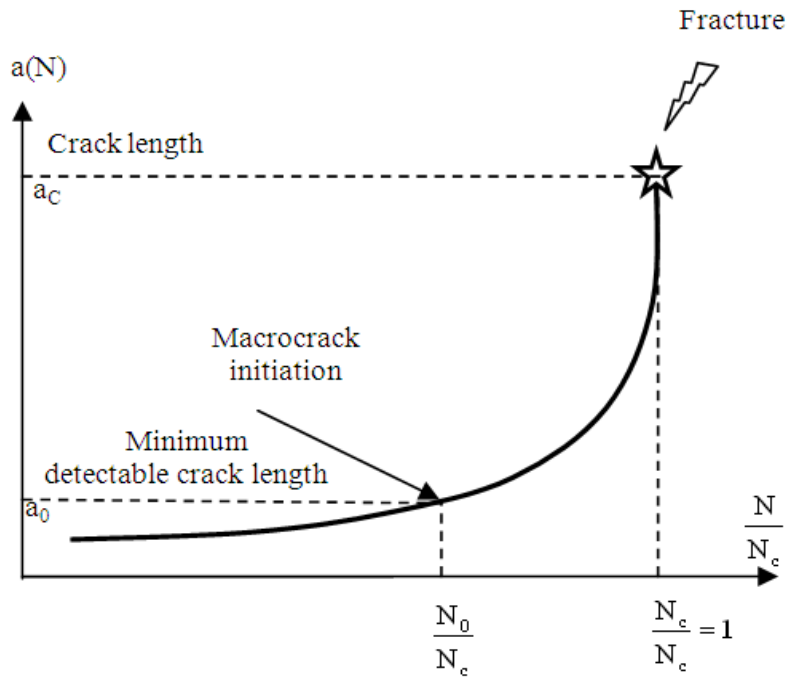


Fig. 2. Pre-Crack fatigue damage

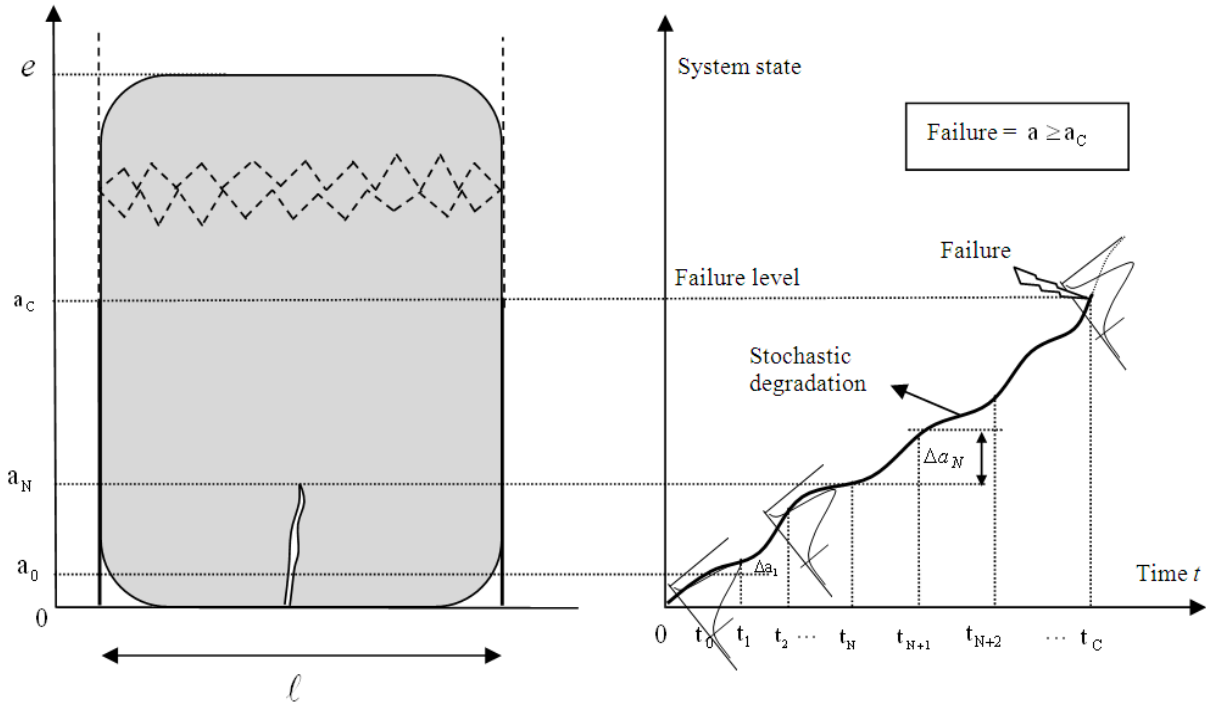


Fig. 3. Probabilistic crack width growth

Where:

$$\Delta K(a) = Y(a) \cdot \Delta \sigma \cdot \sqrt{\pi \cdot a}$$

If we integrate the two sides between initial state N_0 and an arbitrary state N , we get Equation 5:

$$\int_{a_0}^{a_N} \frac{da}{[\Delta K(a)]^m} = \int_{N_0}^N C \cdot dN \quad ; \quad (N_0 = 0) \Rightarrow$$

$$\int_{a_0}^{a_N} \frac{da}{[\Delta K(a)]^m} = \int_{a_0}^{a_N} \frac{da}{[Y \cdot \Delta \sigma \cdot \sqrt{\pi \cdot a}]^m} \tag{5}$$

$$= \frac{1}{Y^m \cdot (\Delta \sigma)^m \cdot \pi^{m/2}} \int_{a_0}^{a_N} \frac{da}{a^{m/2}}$$

From (4) and (5), we get the following Equation 6-7:

$$\Rightarrow a_N = \left[a_0^{1-m/2} + N.C. \left(1 - \frac{m}{2}\right) \cdot (Y \cdot \Delta \sigma \cdot \sqrt{\pi})^m \right]^{\frac{2}{2-m}} \tag{6}$$

And:

$$a_0 = \left[a_N^{1-m/2} - N.C. \left(1 - \frac{m}{2}\right) \cdot (Y \cdot \Delta \sigma \cdot \sqrt{\pi})^m \right]^{\frac{2}{2-m}} \tag{7}$$

As: $a_0 \leq a_N \leq a_C$, then, if we have the PDF of a_0 : $f_0(a_0)$, thus we can deduce the PDF of a_N and a_C as follows:

$$f_0(a_0) \xrightarrow{\text{Jacobian}} f_N(a_N) \xrightarrow{\text{Jacobian}} f_C(a_C)$$

Initial state ($N_0 = 0$) Arbitrary state (N) Critical (final) state (N_C)

Then, the following transformation can be given Equation 8:

$$f_N(a_N) = f_0(a_0) \times |J| = f_0(a_0) \times \left| \frac{da_0}{da_N} \right| \tag{8}$$

with the Jacobian $J = \frac{da_0}{da_N}$ Equation 7-12:

$$f_0(a_0) = f_0 \left[\left[a_N^{1-m/2} - N.C. \left(1 - \frac{m}{2}\right) \cdot (Y \cdot \Delta \sigma \cdot \sqrt{\pi})^m \right]^{\frac{2}{2-m}} \right] \tag{9}$$

Let:

$$\beta = 1 - \frac{m}{2} \Rightarrow m = 2(1 - \beta)$$

And:

$$\frac{m}{2 - m} = \frac{1 - \beta}{\beta} \quad \text{and} \quad \frac{2}{2 - m} = \frac{1}{\beta}$$

$$\Rightarrow f_0(a_0) = f_0 \left[\left[a_N^\beta - N.C.\beta \cdot (Y \cdot \Delta \sigma \cdot \sqrt{\pi})^{2(1-\beta)} \right]^{\frac{1}{\beta}} \right] \tag{10}$$

$$\Rightarrow f_0(a_0) = f_0 \left[\left(a_N^\beta - N.A \right)^{\frac{1}{\beta}} \right] \tag{11}$$

Where:

$$A = C.\beta \cdot (Y \cdot \Delta \sigma \cdot \sqrt{\pi})^{2(1-\beta)}$$

Then:

$$\frac{da_0}{da_N} = \frac{d}{da_N} \left[\left(a_N^\beta - N.C.\beta \cdot (Y \cdot \Delta \sigma \cdot \sqrt{\pi})^{2(1-\beta)} \right)^{\frac{1}{\beta}} \right] \tag{12}$$

Therefore, the Jacobian is given by Equation 13:

$$\frac{da_0}{da_N} = a_N^{\beta-1} \times \left(a_N^\beta - N.A \right)^{\frac{1-\beta}{\beta}} \tag{13}$$

From Equation (8) and (11), the PDF of a_N is given as follows Equation 14:

$$f_N(a_N) = f_0 \left[\left(a_N^\beta - N.A \right)^{\frac{1}{\beta}} \right] \times \left| a_N^{\beta-1} \cdot \left(a_N^\beta - N.A \right)^{\frac{1-\beta}{\beta}} \right| \tag{14}$$

1.6. PDF of the Initial Damage D_0

The relationship between the initial crack length a_0 and the initial damage D_0 is as follows Equation 15-16:

$$D_0 = \frac{a_0}{a_C - a_0} \Rightarrow a_0 = \frac{a_C \cdot D_0}{1 + D_0} \tag{15}$$

The probabilistic transformation theory gives:

$$f_0(D_0) = f_0(a_0) \times \left| \frac{da_0}{dD_0} \right|$$

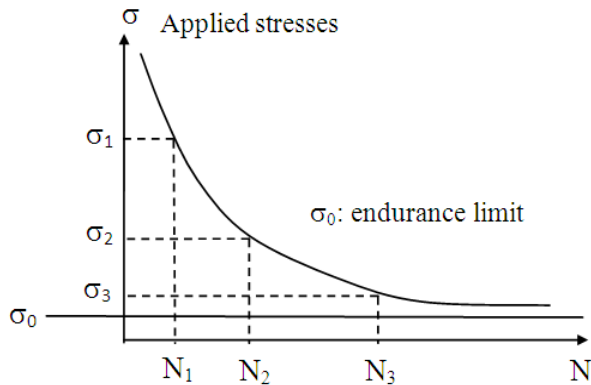


Fig. 4. Wöhler curve of fatigue

As:

$$\frac{da_0}{dD_0} = \frac{a_c}{(1+D_0)^2} \geq 0$$

$$\Rightarrow f_0(D_0) = f_0(a_0) \times \frac{a_c}{(1+D_0)^2} \tag{16}$$

If the proposed law for a_0 is Log-Normal, then the law of D_0 is also Log-Normal with the following PDF:

$$f_0(D_0) = \frac{1}{a_0 \xi \sqrt{2 \cdot \pi}} \exp \left[-\frac{1}{2 \cdot \xi^2} (\ln(a_0) - \lambda)^2 \right] \times \frac{a_c}{(1+D_0)^2}$$

As:

$$a_0 = \frac{a_c D_0}{1+D_0} \quad \text{and} \quad a_c = e/8$$

Then, we can write the PDF as follows Equation 17:

$$f_0(D_0) = \frac{1}{\xi \sqrt{2 \cdot \pi}} \exp \left[-\frac{1}{2 \cdot \xi^2} (\ln[G] - \lambda)^2 \right] \times \frac{1}{F} \tag{17}$$

With:

$$G = \frac{eD_0}{8(1+D_0)} \quad \text{and} \quad F = D_0(1+D_0)$$

1.7. Equation of the Stochastic-Based Prognostic

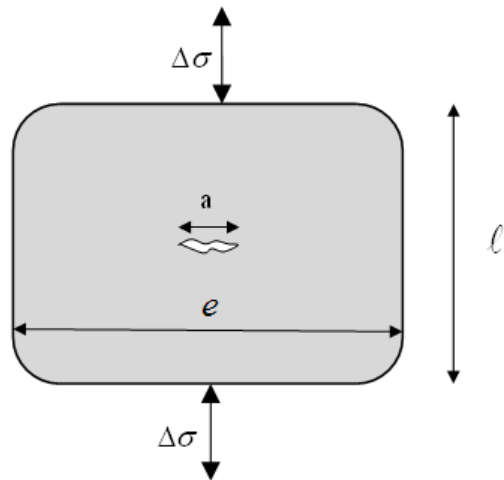


Fig. 5. Critical crack length "a" perpendicular to stress loading

The stress range in fatigue is governed by the Wöhler curve (Lemaitre and Chaboche, 1994) (**Fig. 4**). The transversal crack is critical when it is normal to the stress loading σ (**Fig. 5**).

In case of pipes of thickness e and radius R , the stress ranges $\Delta\sigma$ are created by the applied internal pressure; hence, the following relation gives the critical hoop stress range $\Delta\sigma_\theta$ in terms of pressure range ΔP from the mechanics of materials and shell theory (Timoshenko and Woinowsky-Krieger, 1959) (**Fig. 5 and 6**) Equation 18:

$$\Delta\sigma_\theta = 2 \cdot \Delta\sigma_L = \frac{\Delta P \cdot R}{e} \tag{18}$$

The simulation of internal pressure following a Triangular law generates a sample of stress ranges $\Delta\sigma$ following the same Triangular law.

From Equation (4), where:

$$\Delta K_I(a)$$

is the stress intensity factor (Timashev *et al.*, 2005) and:

$$Y(a) = 0.6 \times \frac{1+2(a/e)}{(1-a/e)^2}$$

is the geometric function:

$$\Rightarrow \Delta K_I(a) = 0.6 \times \frac{1+2(a/e)}{(1-a/e)^2} \times \sqrt{\pi \cdot a} \times \frac{\Delta P \cdot R}{e} = \sqrt[m]{\frac{da}{C \cdot dN}}$$

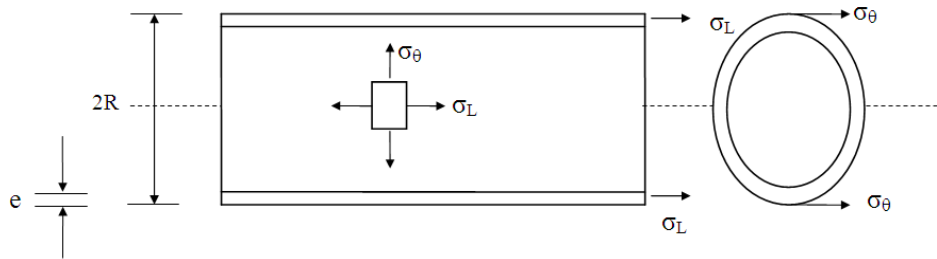


Fig. 6. Stresses in cylindrical pipelines

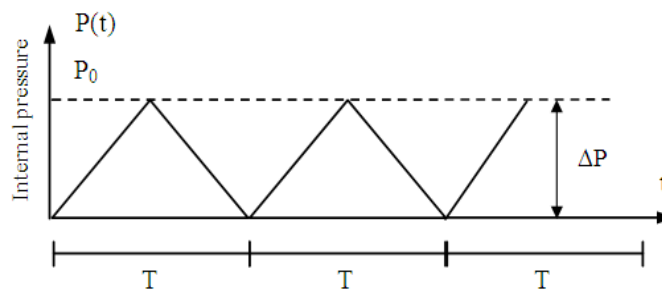


Fig. 7. Triangular pressure law

Then, at the end of each one loading cycle ($dN = 1$) where $i = N$, we can write Equation 19:

$$0.6 \times \frac{1 + 2(a_i / e)}{(1 - a_i / e)^2} \times \sqrt{\pi \cdot a_i} \times \frac{\Delta P_i \cdot R}{e} = \sqrt[m]{\frac{a_i - a_{i-1}}{C}} \quad (19)$$

In earlier work (Abou Jaoude *et al.*, 2010), the degradation evolution is deduced in terms of crack length growth for each cycle i and is cited in Equation (1). Then Equation 20:

$$dD_i = \frac{da_i}{a_c - a_0} = \frac{a_i - a_{i-1}}{a_c - a_0} \quad (20)$$

$$\Rightarrow a_i - a_{i-1} = dD_i (a_c - a_0) = dD_i (e/8 - a_0)$$

Knowing that $1.01 \leq e/a \leq 10$ and $e/a_c = 8$.

From Equation (19) and (20) ($\Delta P_i = P_i - 0 = P_i$, refer to Fig. 7) the degradation increment can be given by the following form:

$$dD_i = \frac{C}{(e/8 - a_0)} \times \left(0.6 \times \frac{1 + 2(a_i / e)}{(1 - a_i / e)^2} \times \sqrt{\pi \cdot a_i} \times \frac{P_i \cdot R}{e} \right)^m \quad (21)$$

and $D_i = D_{i-1} + dD_i$

The previous relation describes the degradation evolution in terms of the following variables: initial crack size a_0 , internal pressure at cycle i which is P_i and the current crack size a_i . This relation represents the stochastic recursive prognostic model as it permits to relate the degradation indicator D_i to the basic random variables (a_0, P_i).

At each loading cycle ($0 \leq i = N \leq N_c$), the degradation indicator D_i increments of a quantity dD_i starting from $D_0 = 0$ till reaching unit value ($D_c = 1$) that is the failure state. Equation (21) represents the stochastic degradation state at cycle i .

Parameters C and m are variables depending on the environment (temperature, humidity, soil action, applied load location, body shape) and the material properties (brittle, ductile plastic, toughness). C and m depend on the testing conditions, such as loading ratio $\sigma_{min}/\sigma_{max}$, on the geometry and size of the specimen and on the initial crack length. They are evaluated by the mean of experiments in true conditions. For these reasons, these parameters, estimated from experimental measurements, are given by numerical samples and hence they can be taken as random variables with mean and standard deviation values. The influence of these parameters on D is shown very clearly in Equation (21).

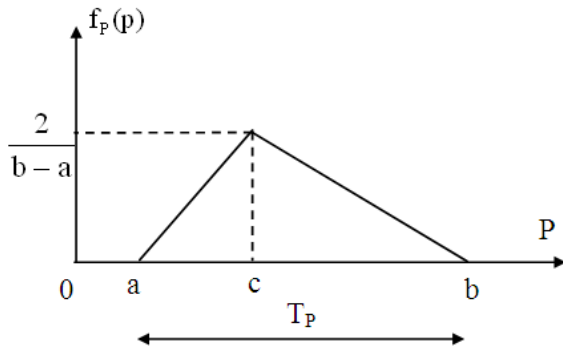


Fig. 8. Triangular PDF function of P

In terms of crack width, the degradation is expressed by a crack increment at the end of each loading cycle ($dN = 1$) by the following recursive relation Equation 22:

$$da_i = C \cdot \left(Y(a_i) \times \sqrt{\pi a_i} \times \Delta \sigma_{\theta} \right)^m$$

$$= C \cdot \left(0.6 \times \frac{1 + 2(a_i / e)}{(1 - a_i / e)^2} \times \sqrt{\pi a_i} \times \frac{P_i R}{e} \right)^m \quad (22)$$

and $a_i = a_{i-1} + da_i$
 (at $i = 0$ then $a_i = a_0 =$ the initial crack length)

1.8. Stochastic RUL

The estimated RUL is then no longer deterministic, but affected by some risk percentage in order to be realized, hence, a bundle of RULs trajectories can be plotted.

Knowing that the RUL function can be expressed by various forms like for example in fatigue mechanics by: crack length a_c , or critical number of cycles N_c , or material tenacity K_{IC} depending on the chosen limit states: service limit state ($a \leq a_c$), or lifetime limit state ($N \leq N_c$), or strength limit state ($K \leq K_{IC}$).

The RUL adopted in this work is the lifetime limit state: N_c -N in terms of the number of loading cycles which can be easily converted to time t.

1.9. Generation of Internal Pressure P

In this case the internal pressure is simulated by the Monte Carlo method along a Triangular distribution over one period T_P (Fig. 8). The choice of this distribution is explained by the fact that during fluid alimentation, the pumping in tubes follows a linear increase and when pumping stops, the internal pressure decreases linearly.

The Triangular law of the internal pressure is given by the following functions.

The Probability Density Function (PDF) of internal pressure P is Equation 23:

$$f_p(p) = \begin{cases} \frac{2(P-a)}{(b-a)(c-a)} & a \leq P \leq c \\ \frac{2(b-P)}{(b-a)(b-c)} & c \leq P \leq b \\ 0 & P < a \text{ and } P > b \end{cases} \quad (23)$$

The Cumulative Density Function (CDF) of internal pressure P is Equation 24:

$$F_p(p) = \begin{cases} 0 & P < a \\ \frac{(P-a)^2}{(b-a)(c-a)} & a \leq P \leq c \\ 1 - \frac{(b-P)^2}{(b-a)(b-c)} & c < P < b \\ 1 & P \geq b \end{cases} \quad (24)$$

The inverse of the CDF function gives a realization for P as follows Equation 25:

$$P = F^{-1}(u) = \begin{cases} a + \sqrt{u(b-a)(c-a)} & 0 \leq u \leq \theta \\ b - \sqrt{(1-u)(b-a)(b-c)} & \theta \leq u \leq 1 \end{cases} \quad (25)$$

where, u is the uniform-based generated value in interval [0,1]:

$$\text{Take: } \theta = \frac{c-a}{b-a}$$

Then, the mean value:

$$\bar{P} = \frac{(1-\theta^3)}{6(1-\theta)} \approx \frac{a+c+b}{3}$$

And the variance Equation 26:

$$\text{var}(P) = \frac{1-\theta \times (1-\theta)}{18} = \left(\frac{b-a}{18} \right)^2 \times \left\{ 1 - \frac{(c-a)(b-c)}{(b-a)^2} \right\} \quad (26)$$

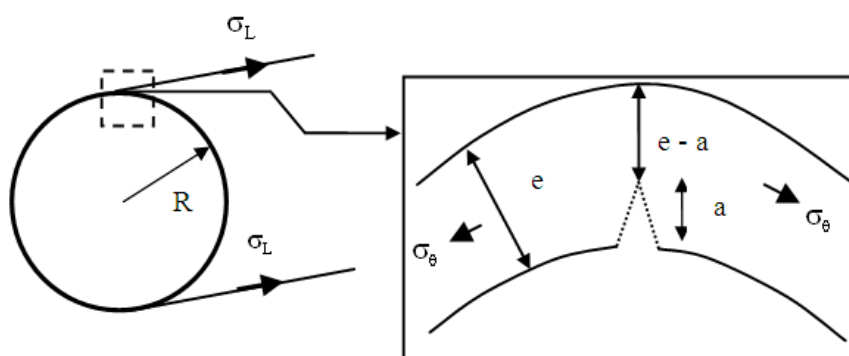


Fig. 9. Cracked pipe section

1.10. Simulation Procedure

Here, the simulation of the internal pressure is done using $F^{-1}(u)$ along one interval T_p under a Triangular law of mean value \bar{P} Equation 27:

$$\bar{P} \approx \frac{0 + P_0 + T_p}{3} \quad (27)$$

For the same initial period, each simulation gives a different realization of the PDF; thus, a new value for $c = P_0$ is given, keeping always $a = 0$ and $b = T_p$.

The following values for the simulation are considered (Fig. 8): $a = 0$, $b = T_p$ (pressure interval: taken as a percentage of P_0) and $c = P_0$ (pressure value: Table 1).

1.11. Prognostic Study of Pipelines Systems

Pipelines are principal components in petrochemical industries and systems. They serve to transport oil and natural gas from plants to consumers' sites. Their life prognostic is vital in this industry since their availability has crucial consequences on the exploitation cost. The main failure cause for these systems is the fatigue due to internal pressure-depression variation along the time. Usually, three situations for these pipes exist: unburied, buried and under sea water (offshore pipes). The study of each one of these situations requires data about some physical parameters like: corrosion, soil pressure and friction (for buried pipes), water and atmospheric pressure (for offshore pipes).

The present work is limited to the unburied case. This case (Sukumar *et al.*, 2003) is suitable outside cities between states and countries where they do not intercept any construction or transportation facilities. In this case, the normal service load includes only the internal pressure beside the environmental effects.

Table 1. Three pressure modes

Pressure mode	P_0 (MPa)
High (mode 1)	8
Middle (mode 2)	5
Low (mode 3)	3

1.12. Mechanical Modeling

The pipes are cylindrical thin tubes since their thickness e to radius ratio R is (Abou Jaoude *et al.*, 2011b): $e/R \leq 1/10$. In this case, the membrane stresses without any bending forces are the circumferential (hoop stress) σ_θ and at right angle, the longitudinal (axial stress) σ_L given by Equation 28:

$$\sigma_\theta = \frac{P \cdot R}{e}; \quad \sigma_L = \frac{P \cdot R}{2 \cdot e} \quad (28)$$

The critical position of cracks is longitudinal which is perpendicular to the direction of maximal stresses σ_θ . It has a depth (or length) a measured in the direction of tube thickness e (Fig. 9). Generally, it can be considered the following ratio interval: $0.1 \leq a/e \leq 0.99$.

Consider a pipe of radius $R = 240$ mm and of thickness $e = 8$ mm transporting natural gas. In this case, the material and environment parameters are $C = 5.2 \cdot 10^{-13}$ (free air) and $m = 3$ (metal).

1.13. Simulations of Three Levels of Internal Pressure

Three maximal levels of $P = P_0$ are considered (Table 1) with a repetition period T_p . For each of these levels, a degradation trajectory D is deduced in term of cycle number N . when D reaches the unit value, then the corresponding N is called the lifetime of the pipe that failed by fatigue.

1.14. Basic Random Variables

The generation of the two basic random variables (a_0 and P) is done along the Log-Normal and the Triangular laws respectively.

The initial crack length a_0 is simulated along the Log-Normal law with the following parameters:

$$a_0 : \text{Log - Normal law} \begin{cases} E(a_0) = 0.2 \text{ mm} \\ \sigma_{a_0} = \sqrt{\text{Var}(a_0)} = 0.0029 \text{ mm} \end{cases}$$

Then, the equivalent Normal parameters for a_0 are:

$$\lambda = -1.6095 \quad ; \quad \xi = 0.014724$$

The internal pressure is simulated along a Triangular law for three different modes P_0 (**Table 1**).

With mean value:

$$\bar{P} \approx \frac{0 + P_0 + T_p}{3}$$

We can deduce from Equation (26) the variance as follows:

$$\text{var}(P) = \frac{(T_p)^2 - (P_0)(T_p - P_0)}{18^2}$$

2. RESULTS

The Triangular simulation of the pressure inside the unburied pipes and for the 3 modes leads to the applied stress blocks shown in **Fig. 10** below. This figure shows that, for the 3 blocks of applied stresses, the randomness is clearly illustrated by the fluctuation values of these stresses with the cycle numbers. The mean values of the 3 blocks are respectively 240, 150 and 90 MPa.

The crack width $a(t)$ growth versus time is given in **Fig. 11** that shows the width evolution from an initial value a_0 to the critical value $a_c = e/8$. The crack widths versus time are given in this figure for the 3 modes. They grow from an initial value $a_0 = 0.2 \text{ mm}$ to the end of life where all curves $a(t)$ reach the critical width $a_c = e/8 = 1$. The high pressure mode reveals the fastest width increase.

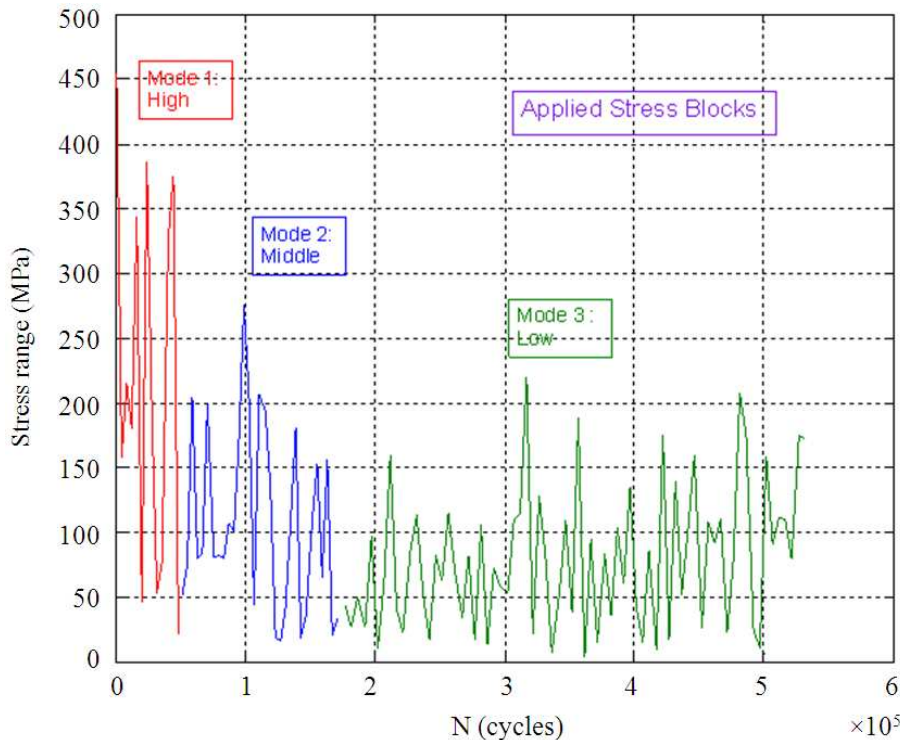


Fig. 10. Applied stress blocks in pipes for 3 modes of pressure

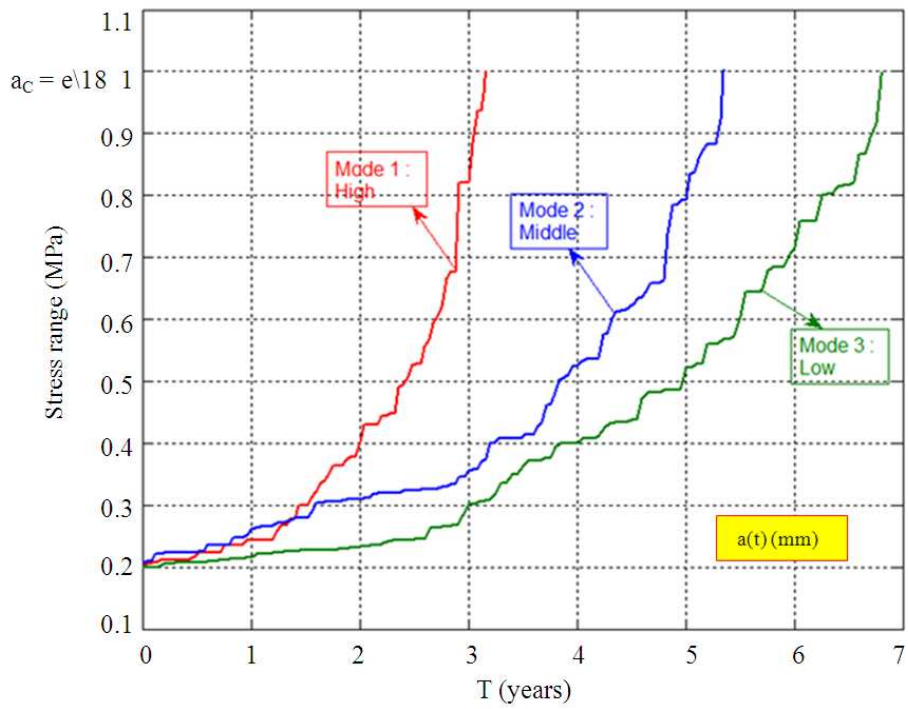


Fig. 11. Crack width evolution with time for 3 modes of pressure

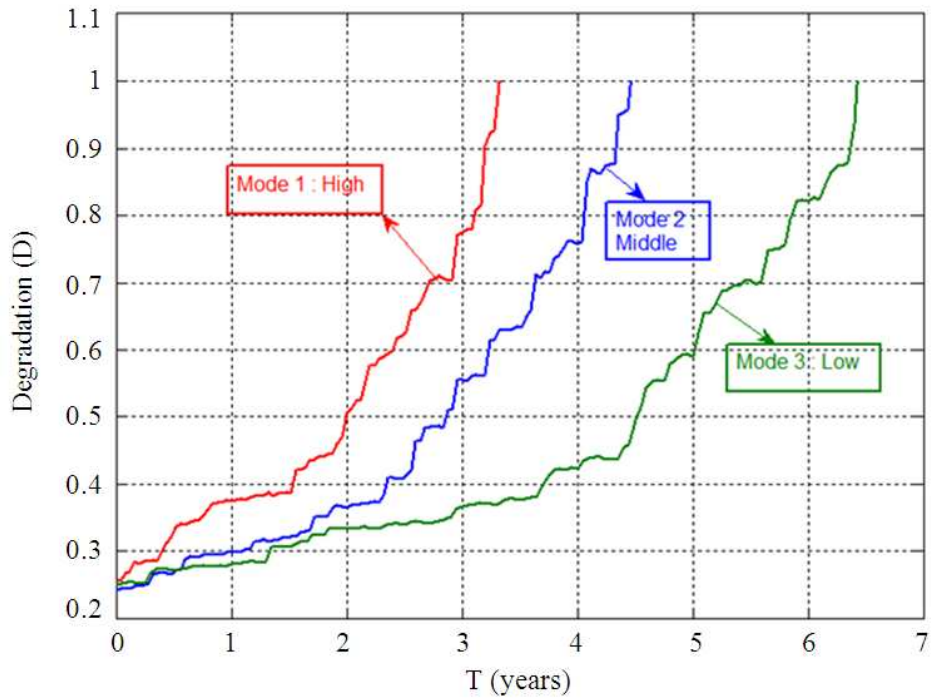


Fig. 12. Pipe degradation evolution for 3 modes of pressure

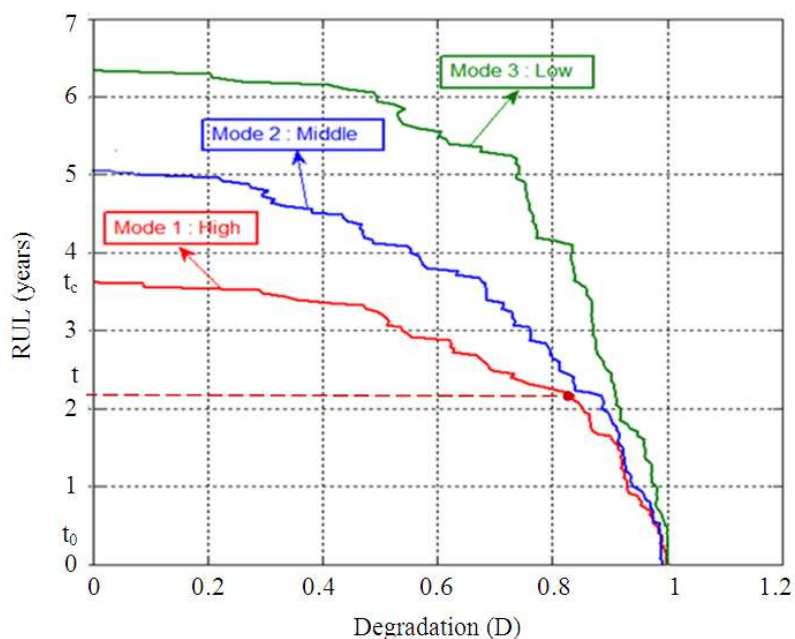


Fig. 13. Pipe RUL evolution for 3 modes of pressure

We should note here that even though "a" and D appear to be identical in the simulation curves, they are different because "a" varies generally from a_0 to a_c (that happens to be here = 1 since $e = 8$ mm) whereas D varies in all cases from 0 to 1 and it is then a more convenient index for damage measurement since it is normalized.

The simulation of the prognostic Equation (21) previously developed permits to draw, for each level of pressure (high, middle, low), the degradation trajectory D in terms of time t.

The results of degradation trajectory simulations are shown in Fig. 12 below. The degradation indicator D evolves from 0 to 1 (end of its life) and this for each pressure mode. The obtained lifetimes values are verified to be in the range of real lifetimes according to the references (Guan *et al.*, 2010; Xiang and Liu, 2010).

The pipe lifetimes for this case are fixed when D reaches the unit value. They are nearly 3.36 years for mode 1 (high pressure), 4.48 years for mode 2 (middle pressure) and 6.45 years for mode 3 (low pressure).

Conversely, at each instant t, the $RUL(t) = t_c - t$ (Fig. 13) can be deduced starting from the raw state of the pipe $RUL(t_0) = t_c - t_0$ which gives the entire age of the pipe, till reaching the failure state ($D = 1$) where $RUL(t_c) = t_c - t_c = 0$ (Fig. 13 for Mode 1: High).

As it can be noticed, these curves are stochastic and the lifetimes deduced from them are also stochastic.

Therefore, we do not have a unique value for the corresponding RUL, but rather a new realization is derived from each simulation and then a mean value of RUL can be inferred.

The stochastic influence can be seen through the variability over the curve realizations of $\tilde{D}(N)$ obtained by several simulations and not from just one realization.

The mean curve $\bar{D}(N)$ is plotted from the mean value of these realizations. The conservative curves are those that give the maximum values. For each mode, the characteristic value of lifetime can be computed from the mean value, the standard deviation and a certain fractal percentage depending on the risk adopted by specialized decision makers.

2.1. Flowchart of the Stochastic-Based Linear Prognostic

A flowchart given in Fig. 14 below summarizes all the executive steps of the proposed model. It is divided into two main parts: the stochastic part where the simulations of variables are done and the damage accumulation part where the measure of degradation D and crack length a are updated and cumulated after each cycle N.

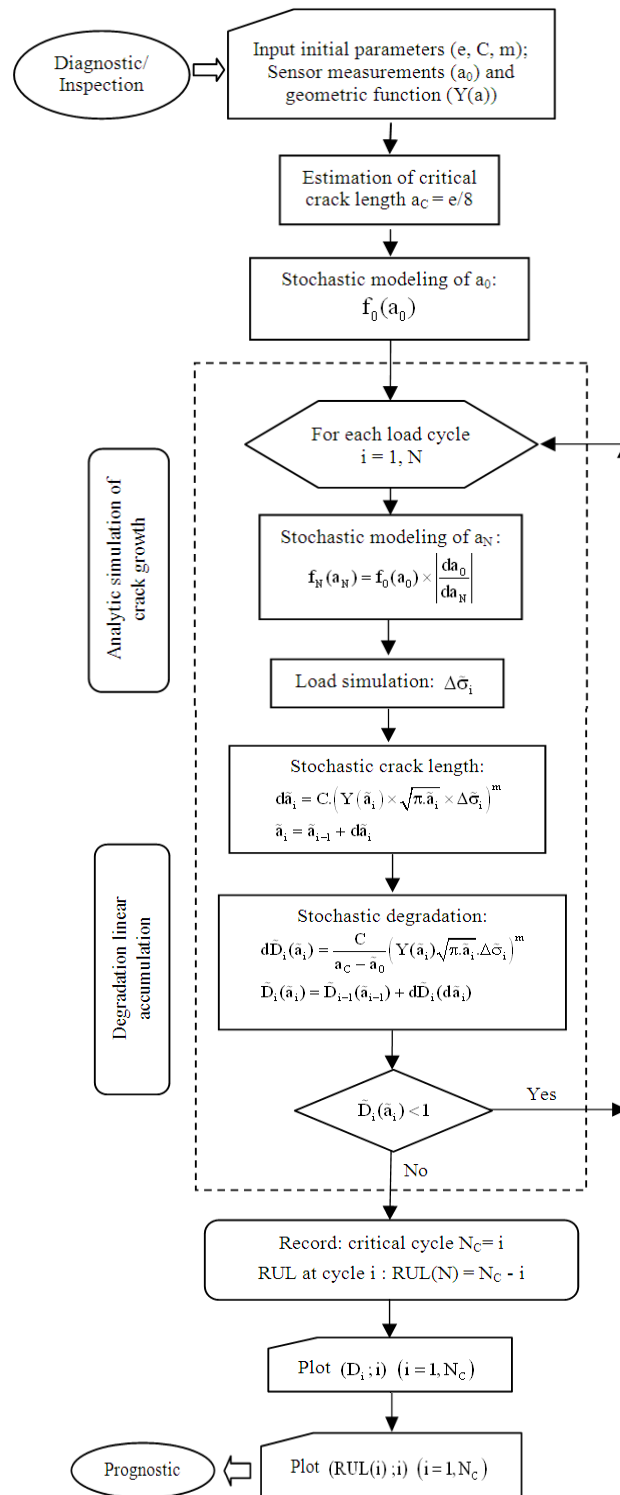


Fig. 14. Flowchart of the proposed model

3. CONCLUSION

A prognostic model is proposed in this work based on the accumulation of damage due to fatigue crack propagation in stochastic conditions. The damage state of the device is measured by a degradation indicator in terms of the number of loading cycles. To show the efficiency of this prognostic model, it is applied to predict the fatigue life of an unburied pipeline under 3 modes of pressure where the initial crack length and the internal pressure are taken random.

The main differences between the degradation measurement D and the crack length a are: (1) D is normalized (from 0 to 1) while a is not generally (from a_0 to a_c); (2) D can take into account many damage parameters (corrosion, cracks, deflection, resonance) while a represents only one damage parameter which is the crack length itself.

As a prospective work, it is planned to explore the variability of the stochastic lifetimes in order to deduce a characteristic one attached to some acceptable risk.

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