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On the Stability of a Certain Class of Linear Time-Varying Systems

M. De la Sen
Instituto de Investigacion y Desarrrollo de Procesos IIDP, Facultad De Ciencias
Universidad del País, Vasco, Leioa (Bizkaia), Aptdo, 644 de Bilbao, Spain

Abstract: This study firstly considers the exponential stability of unforced linear systems of slowly time-varying dynamics. Possible switchings of the system structure to unstable dynamics during certain finite time intervals are admitted. The maintenance of global exponential stability does not necessarily require at most a finite number of switchings in the dynamics while infinitely many switches can also lead to stability. The mechanism to achieve stability under infinitely many switches in the dynamics is to maintain the system in the stable region during time intervals of sufficient large length without switches provided that the system dynamics evolves at a sufficiently small rate with time. Special attention is paid to the robust tolerance for a class of state disturbances and to the case of time-varying matrix of dynamics that possess either piecewise constant or constant eigenvalues. The obtained results can be relevant for their use in stability issues for the cases of multimodel non- adaptive and adaptive control with improved transient performances.

Key words: Bohl transformations, exponential stability, time-varying linear systems

INTRODUCTION

It is well known that unforced piecewise - constant linear systems, whose associated matrix of the dynamics takes values in a set of strictly Hurwitzian matrices are not guaranteed to be exponentially stable[1-3]. Instability can occur when the infinity of switches between elements of that set is performed. However, exponential stability is preserved when the number of switches is finite or when infinitely many switches occur while all the set of Hurwitzian matrices admits similar upper or lower triangular forms under the same transformation matrix.

The above last situation has been investigated[3] and pointed out to be very restrictive in the sense that only systems being direct extensions of decoupled scalar and/or second- order systems can be considered. A known surprising result is that time- varying systems with constant strictly stable eigenvalues may be unstable if the parameters of the dynamics matrix do not vary at a sufficiently small slope[4]. The problem of switching operations between configurations of piecewise continuous stable dynamics is of growing interest in multimodel design with improved transient performances. The related problem of time- varying dynamics of piecewise constant eigenvalues are of relevant interest in adaptive control. In this case, the use of switches between several reference models is a useful tool to improve the adaptation transient performances.

In this study, the stabilization problem is focused on by keeping a slow time- varying system with a stable dynamics during (non-necessarily consecutive) so called stabilization time intervals. These intervals compensate for possible large deviations of the equilibrium along time for the final time of each last preceding stabilization interval. A time-scheduling rule is designed for switching from a controller to another possible one, within a prefixed finite set of stabilizing controllers, while maintaining the global exponential stability. It is also proved that the system exhibits robust exponential stability against state disturbances that vary non faster than linearly with the state at small rates. Furthermore, the system has proven to be robustly stable in terms of ultimate boundedness against a class of non- linear state dependent disturbances when the unforced dynamics have associated stable piecewise constant eigenvalues during certain stabilization time- intervals of sufficiently large lengths.

In the performed stability analysis, it is taken into account the fact that, in general, well- posed transformations of coordinates on linear time- varying systems do not necessarily preserve possible stability properties[4,6]. The mechanism used for establishing the obtained stability results is to describe the system dynamics in a normalized time-varying canonical form whose upper off-diagonal entries are of sufficiently small absolute values compared to some prescribed positive threshold[5]. A Bohl transformation relates the given state- space description to the canonical one whose Jordan matrix is piecewise constant in the same way as the eigenvalues of the original dynamics. Thus, the original description is ensured to be stable if the system exhibits stability in the transformed coordinates since Bohl transformations preserve the stability properties[6].

Notation: $\| \|_2$ denotes the l_2 - matrix or vector norms.

Corresponding Author: M. De la Sen, Instituto de Investigacion y Desarrrollo de Procesos IIDP, Facultad De Ciencias Universidad del País, Vasco, Leioa (Bizkaia), Aptdo, 644 de Bilbao, Spain

Card (.) is the finite or (denumerable) infinity cardinal of the (.)- set.

R and $\mathbf{R}^+$ denote the sets of real and positive real numbers, respectively. $\mathbf{R}_0^+ := \mathbf{R}^+ \cup \{0\}$. A similar notation stands for the sets of integers by replacing **R** to **Z**. Finally, **C** is the set of complex numbers.

sp (.) is the spectrum of the (.)- matrix (i. e., the set of its, in general, complex eigenvalues) ess $\sup_{t \in I}$ (.) is the essential supremum of the real (.)- function on I, namely, $\operatorname{ess}\sup_{t\in I}(.) = \sup_{t \in I_0}(.)$ for any $I_0 \subseteq I$ such that the set I - I_0 is of zero measure.

' > ' and ' ≥ ' stand for definite and semidefinite positive matrices, respectively.

λ_{max} (.) and λ_{min} (.) denote, respectively, the maximum and minimum eigenvalues of the (.)- matrix.

Superscripts T and * stand, respectively, for the transpose and complex conjugate transpose of a vector or matrix.

The notation exp (f) denotes the exponential off and is used, instead of e f, when f is a complex or cumbersome expression.

$\mathbf{S}_\mathbf{A} := \bigcup_{i \in \mathbf{S}} [t_i, t_{i+1})$ is the stabilization time interval, namely the, in general, the disjoint union of time connected subintervals $[t_i, t_{i+1})$ (i.e., the connected components of $\mathbf{S}_\mathbf{A}$), $i \in \mathbf{S}$ ($\mathbf{S} \subseteq \mathbf{Z}^+$ being the stabilization indicator set) of durations or lengths (or, more rigorously speaking, measures) given by the switching time- scheduling rule where the matrix of dynamics A (t) has eigenvalues of strictly negative real parts.

D, $\overline{\mathbf{D}}$ and **D** (t_a, t_b) are the sets of discontinuity times of the matrix of dynamics A (t) on the stabilization interval $\mathbf{S}_\mathbf{A}$, its complementary $\overline{\mathbf{S}}_\mathbf{A}$ and the connected time interval $[t_a, t_b)$, respectively. $\mathbf{D}(t_i, t_{i+1})$ is simply denoted by $\mathbf{D}_\mathbf{i}$ for consecutive times t_i being members of a sequence of time instants $\{t_i, i \geq 1\}$ as, for instance, when $[t_i, t_{i+1}) \subseteq \mathbf{S}_\mathbf{A}$ for $i \in \mathbf{S}$.

STABILITY RESULTS FOR SLOWLY TIME-VARYING LINEAR SYSTEMS

Consider the homogeneous linear time-varying dynamic system:

$$\dot{x}(t) = A(t)x(t); x(0) = x_0 \in \mathbf{R}^n \qquad (1)$$

where, A: [0, ∞) $\to \mathbf{R}^{n \times n}$. The subsequent stability result holds:

Theorem 1: Consider the system (1) under the following assumptions.

A.1: A (t) is bounded and of piecewise continuous entries which are also time-differentiable for all t ≥ 0 where such time-derivatives exist (i.e., $\mathbf{R}_0^+$ excluding all possible isolated discontinuity points).

A.2: $\int_t^{t+T} \|\dot{A}(\tau)\|_2^2 d\tau \leq K_{dA}^2(t,t+T)T + K'_A(t,t+T)$ with in general, time-interval dependent constants $K_{dA}(t, t+T) < \infty$; $0 \leq K'_A(t, t+T) < \infty$ for all t ≥ 0 and all finite T ≥ 0.

A.3: There exists a non empty and (in general) non connected stabilization time interval of infinite measure $\mathbf{S}_\mathbf{A} := \bigcup_{i \in s} [t_i, t_{i=1})$, where $\mathbf{S} \subseteq \mathbf{Z}^+$ is the stabilization indicator set (i. e., the indicator set of the stabilization interval) which consist of the (in general disjoint) countable union of connected time intervals $[t_i, t_{i+1})$, i ≥ 1, such that:

1. The eigenvalues of A (t) are strictly inside the stability boundary for all $t \in \mathbf{S}_\mathbf{A}$.
2. Assumption A.2 holds for all bounded time interval $[t_i, t_{i+1}) \subseteq \mathbf{S}_\mathbf{A}$ $(i \in \mathbf{S})$ with sufficiently small $K_{dA}(t_i, t_i + \tau)$, all $\tau \in [t_i, t_{i+1}]$. If there is an interval $[t_{i_s}, \infty) \subseteq \mathbf{S}_\mathbf{A}$ of infinite measure (i. e. Card (**S**) = s < ∞ so that t_{i_s} is finite) then Assumption A.2 holds for any finite or infinite time interval $[t, t+T] \subseteq [t_i, \infty)$ with $i \in \mathbf{S}$.

There is at most a finite set $\mathbf{D}_\mathbf{i}$ of discontinuities of A(t) within each stabilization subinterval $[t_i, t_{i+1}) \subseteq \mathbf{S}_\mathbf{A} (i \in \mathbf{S})$ of finite or infinite measure (with eventual switches being subject to Assumption A.2.1). The discontinuities satisfy t '' ≥ t ' + T^* for any prefixed finite $T^* > 0$ and for all t ', t'' $\in \mathbf{D}_\mathbf{i}$ with t ''> t ' and $i \in \mathbf{S}$.

There is at most a finite set $\overline{\mathbf{D}}$ of discontinuities of A (t) in $\overline{\mathbf{S}}_\mathbf{A} := \mathbf{R}_0^+ - \mathbf{S}_\mathbf{A}$ with ∞ > (t ''- t') $\geq T^* > 0$ for any two consecutive discontinuities t ', t '' $\in \overline{\mathbf{D}}$.

Thus, the system (1) is globally exponentially stable provided that either:

a. Card (**S**) = s < ∞; i. e., $[t_{i_s}, \infty) \subseteq \mathbf{S}_\mathbf{A}$ for some arbitrary finite t_{i_s}, or

b. $\mathbf{S}_\mathbf{A} = \bigcup_{i \in \mathbf{S}} [t_i, t_{i+1})$ with Card (**S**) = timesnsists of infinitely many stabilization time subintervals of finite lengths $T_i = t_{i+1} - t_i \geq \mathrm{Max}\left(T_i^{'}, T^*\right)$ for some appropriate sequence $\left\{T_i^{'} > 0, i \in \mathbf{S}\right\}$.

The perturbed system $\dot{x}(t) = A(t)x(t) + F(t, x(t))$ is robustly globally exponentially stable if $\left\| F(t, x(t)) \right\|_2 \leq f . \left\| x(t) \right\|_2$ for a sufficiently small positive real constant f.

Comments 1: Theorem 1 is much more powerful than related previous results used[7,8] because of the following features:

a. A (t) is allowed to possess critically stable and even unstable, eigenvalues on finite time intervals. The cases $t \in [t_i, t_{i+1}) \subseteq \mathbf{S}_{\mathbf{A}}$ with A (t) = A_i (strictly Hurwitzian) and sp (A (t))= {$\lambda_j \in \mathbf{C}$: j = 1, 2,..., σ } with Card (**S**) = ∞ are included in Theorem 1 where **S** is the indicator set of $\mathbf{S}_{\mathbf{A}}$ (i. e.,$i \in \mathbf{S} \Rightarrow [t_i, t_{i+1}) \subseteq \mathbf{S}_{\mathbf{A}}$. In the formulation given[1,7,8] only the case Card (**S**) < ∞ with $t_{ij} < \infty$ of Theorem 1 was considered, i. e., a finite set of switches between different dynamics. In this context, the proposed formalism is useful for its potential application in multimodel or adaptive multimodel design with possible infinitely many switches. Note that the multimodel design is a powerful tool to improve the transient behaviors. Note also that it is required for the stabilization interval $\mathbf{S}_{\mathbf{A}}$ to be of infinite measure. Thus, either $[t_{i_s}, \infty) \subseteq \mathbf{S}_{\mathbf{A}}$, some $t_{i_S} < \infty$ and Card (**S**) = s < ∞, or $\mathbf{S}_{\mathbf{A}}$ consists of the infinite countable union of disjoint connected time subintervals [t_i, t_{i+1}) of appropriate finite lengths $T_i = t_{i+1} - t_i$, which depend on the system parameterization and previous interval lengths on $\bar{\mathbf{S}}_{\mathbf{A}}$ and are obtained in Appendix A (eqns. A.28).

b. A (t) is allowed to possess a finite set of discontinuities, where $\dot{A}(t)$ is impulsive, within each stabilization subinterval $[t_i, t_{i+1}) \subseteq \mathbf{S}_{\mathbf{A}}$ and a finite set within $\bar{\mathbf{S}}_{\mathbf{A}}$ (i. e., out of the stabilization time interval). As a result, the total number of discontinuities of A (t) on $\mathbf{S}_{\mathbf{A}}$ can be infinite since $\mathbf{S}_{\mathbf{A}}$ consists of the infinite union of finite intervals. However, when $[t_{i_s}, \infty) \subseteq \mathbf{S}_{\mathbf{A}}$, some $(t_{i_s} < \infty)$ the total number of discontinuities on $\mathbf{S}_{\mathbf{A}}$ is $\bigcup_{i=1}^{s} \mathrm{Card}\left(\mathbf{D}_{\mathbf{i}}\right) < \infty$.

c. The time- integral of $\left\| \dot{A}(t) \right\|_2^2$ ia sufficiently small linear ratently small linear rate with time only within the stabilization time subintervals. A (t) can have critically stable and/or unstable eigenvalues outside $\mathbf{S}_{\mathbf{A}}$. Furthermore, $ess\ sup_{t \in \bar{S}_A}\left(\left\| \dot{A}(t) \right\|_2\right)$ can be large out of the stabilization time subintervals.

The mechanism that allows the achievement of the stabilization is the maintenance of the system beyond some appropriate minimum time after each $t_i \in \mathbf{S}_{\mathbf{A}}$; i.e., to build a set of stabilization time subintervals of sufficiently large lengths T_i. Such a strategy makes possible to compensate for poor transient behaviors occurring from pervious time intervals out of the stabilization interval. In general, the lengths of the stabilization subintervals should increase as the number of previous discontinuities and the value $\operatorname{esssup}_{t \in \bar{S}_A}\left(\left\| \dot{A}(t) \right\|_2\right)$ increase but they are not dependent on the initial conditions of the system (1). Note also that A (t) can be stable, critically stable or even unstable out of the stabilization interval; i. e., for $t \in \bar{\mathbf{S}}_{\mathbf{A}}$. No specific stabilization strategy is taken on $\bar{\mathbf{S}}_{\mathbf{A}}$ even if eventually (the eigenvalues of A(t) are in the stable region at certain subintervals. However, the global exponential stability is ensured by the proposed strategy of selecting the lengths of the stabilization time subintervals.

Example: First, note that, in general, the stability of a linear time-varying system (1) cannot be judged based on the eigenvalues of A (t). For instance, if:

$$A(t) = \begin{bmatrix} -1 + 1.5\cos^2(t) & 1 - 1.5\sin(t)\cos(t) \\ -1 - 1.5\ \sin(t)\cos(t) & -1 + 1.5\sin^2(t) \end{bmatrix}$$

the (the eigenvalues of A(t) are constant and given by -0.25± $0.25\sqrt{7}\,j$. However, the system (1) is unstable, even in the absence of switchings in its dynamics, since its state transition matrix is $\phi(t, 0) = \begin{bmatrix} e^{0.5t}\cos(t) & e^{-t}\cos(t) \\ -e^{0.5t}\sin(t) & e^{-t}\cos(t) \end{bmatrix}$[4]. Also, the system (1) with the above A (t) is exponentially stable (Assumptions A.1-A.2 of Theorem 1) if:

a. There are no switchings and the eigenvalues (constant or not) of A (t) are strictly stable of all time
b. The ratio of the integral of the square- norm time-derivative of A (t) on any time interval related to the length of such an interval is sufficiently small.

Assume now that the non constant part of A (t) has a very small leading coefficient so that the absolute values of the time- derivatives of its entries are small enough to accomplish with Assumption 2 of Theorem 1. In particular, the above matrix A (t) is replaced with $\begin{bmatrix} -1+0.1\cos^2(t) & 1-0.05\sin(t)\cos(t) \\ -1-0.05\ \sin(2t) & -1+0.1\sin^2(t) \end{bmatrix}$ whose constant stable eigenvalues are -0.95$\pm\ \frac{1}{2}\sqrt{0.03}$ j for all time. Then, the resulting system (1) is exponentially stable and satisfies Assumption A.2 of Theorem 1. If now

$$A(t)=\begin{bmatrix} -1+\varepsilon\cos^2(t) & 1-\frac{\varepsilon}{2}\sin(t)\cos(t) \\ -1-\frac{\varepsilon}{2}\ \sin(2t) & -1+\varepsilon\sin^2(t) \end{bmatrix}=A^*+\tilde{A}(t)$$

with $A^*=\begin{bmatrix} -1 & 1 \\ -1 & -1 \end{bmatrix}$ being constant and Hurwitz and $\tilde{A}(t)=\varepsilon\begin{bmatrix} \cos^2(t) & -\frac{\sin(2t)}{2} \\ -\frac{\sin(2t)}{2} & \sin^2(t) \end{bmatrix}$ with being a positive real constant, it follows by taking norms in the solution of the corresponding system (1) that:

$$\|x(t)\|_2 \le \operatorname*{Sup}_{\tau\in[0,T]}\left(\|x(\tau)\|_2\right) \le \sqrt{2}\,e^{-t}\,\|x(0)\|_2 + 6\varepsilon \operatorname*{Sup}_{\tau\in[0,T]}\left(\|x(\tau)\|_2\right)$$

So that the time-varying system is guaranteed to be globally exponentially stable for provided that no switches in the dynamics take place. Exponential stability is also ensured if the constant parameter ε is replaced by a piecewise constant one $\varepsilon(t)=\varepsilon_i<\bar{\varepsilon}$ for all $t\in[t_i, t_{i+1})$ and any finite or infinite sequence of time instants $\{t_i,\ i\ge 1\}$ such that $t_{i+1}-t_i\ge T^*>0$ for any arbitrary and finite T *. Now, assume that A * is not already constant but it switches between the values $A_1^*=\begin{bmatrix} -1 & 1 \\ -1 & -1 \end{bmatrix}$ (exponentially stable) and $A_2^*=\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$ (unstable) and furthermore, $\tilde{A}(t)=\varepsilon(t)\begin{bmatrix} \cos^2(t) & -\frac{\sin(2t)}{2} \\ -\frac{\sin(2t)}{2} & \sin^2(t) \end{bmatrix}$ with $\varepsilon(t)=\varepsilon_i<\bar{\varepsilon}$ for all $t\in[t_i, t_{i+1})$. The switchings between $A_i^*\ (i=1, 2)$ are assumed to occur in the same sequence of time instants $\{t_i,\ i\ge 1\}$ fulfilling that $t_{i+1}-t_i\ge T^*>0$. Assume that $[t_{i-i_0(i)}, t_{i-i_0(i)+1})$ and $[t_i, t_{i+1})$ are two consecutive time intervals where $A^*=A_1^*$ so that they can be potentially used as stabilization subintervals in the sense of Theorem 1. Assume that at least one switch of the dynamics in- between both times subintervals occurs with $A^*=A_2^*$ and $\varepsilon(t)\le\bar{\varepsilon}$ for all time. Thus, the exponential stability of this system is preserved from Theorem 1 if $t_{i+1}\ge \operatorname{Max}\left(T^*, \sum_{k=i-i_0(i)}^{i-1}(t_{k+1}-t_k)\right)$ for all integer i belong to the indicator set of the stabilization interval (i.e., t_i is the initial time of some connected component of the stabilization time interval) and any arbitrary elapsing time T * > 0.

STABILITY OF A CLASS OF FORCED SYSTEMS WITH PIECEWISE CONSTANT EIGENVALUES

Through this section, the stability of the forced linear time-varying dynamical system:

$$\dot{x}(t)=A(t)\,x(t)+F(t,x(t))+G(t,x(t)\ ;\quad x(0)=x_0\in\mathbf{R}^n \qquad (2)$$

is investigated. It is assumed that the unforced system is exponentially stable according to Theorem 1 with the eigenvalues of A (t) being piecewise constant and

$$\|F(t, x(t))\|_2\le M.\|x(t)\|_2^{\alpha}\ ;\quad \|G(t, x(t))\|_2\le\gamma(t) \qquad (3)$$

some real constants $\infty>M>0$ and $0\le\alpha\le 1$, some (in general state- dependent and nonlinear) real vector functions $F:\mathbf{R}_0^+\times\mathbf{R}^n\to\mathbf{R}^n$, $G:\mathbf{R}_0^+\times\mathbf{R}^n\to\mathbf{R}^n$ of disturbances and some scalar function $\gamma:\mathbf{R}_0^+\to\mathbf{R}_0^+$ fulfilling that $\lim_{t\to\infty}\left(\int_t^{t+\omega}\gamma(\tau)d\tau\right)=0$ for any finite $\omega\in\mathbf{R}^+$. It is shown in the following that Lyapunov's stability still holds if the lengths of the stabilization subintervals are chosen sufficiently larger than those required by Theorem 1. The lengths of the stabilization subintervals in Theorem 1 ensure that the homogeneous version of (2), namely, the unforced system (1), is globally exponentially stable. However, larger interval lengths can be needed for ensuring the stability of (2) due to the presence of perturbations. The Lyapunov' s-type stability results obtained in the sequel are based on the achievement of ultimate boundedness for all possible trajectories of (2). The subsequent result holds.

Theorem 2: Consider the forced system (2) - (3) whose homogeneous part satisfies all the assumptions of Theorem 1. Assume that $s_A<\infty$ is the number of distinct stable configurations of A(t) on $\mathbf{R}_0^+$ but the number of switches among them can be either finite or infinity. Thus, the following two propositions hold:

i. Assume also that all the switchings in its dynamics occur between sets of strictly stable constant eigenvalues; i.e., sp (A (t)) is piecewise constant and consists of strictly stable eigenvalues for all $t\in[t_i, t_{i+1})\subseteq\mathbf{S_A}=\mathbf{R}_0^+\ \ (i\in\mathbf{S}=\{1, 2, ..., s\})$ with $\bar{\mathbf{S}}_{\mathbf{A}}$ being empty.

Thus, the forced system (2)-(3) is Lyapunov' s stable with ultimate boundedness if all the members of the sequence $\{T_i, i \in \mathbf{S}\}$ are sufficiently large compared to their lower thresholds provided by Theorem 1.

ii. If one or more eigenvalues of A(t) are critically stable and /or unstable over some finite subinterval $\overline{\mathbf{S}}_{\mathbf{A}} := \mathbf{R}_{\mathbf{0}}^{+} - \mathbf{S}_{\mathbf{A}}$ then the forced system (2)-(3) is still Lyapunov' s stable with ultimate boundedness for some appropriate sequence of lengths $\{T_i, i \in \mathbf{S}\}$ of the set of disjoint subintervals within $\mathbf{S}_{\mathbf{A}}$ which are possibly larger than those obtained in Theorem 1 for the homogeneous system.

Comments 3: First note that the results in both Theorems 1-2 are of sufficient- type since their proofs are based on the application on Gronwall' s Lemma and Lyapunov' s theory. In that way, stabilization is potentially possible under weaker conditions. However, it is obvious by using contradiction arguments that if there exist infinitely many time subintervals, each of finite length, where the dynamics is unstable, then it is necessary to choose the lengths of the stabilization time subintervals sufficiently large to compensate for the local instability generated by the unstable or critically stable dynamics during preceding time subintervals.

The interval lengths of the stabilization subintervals of Theorem 2 are proving to be at least as large as those requested in Theorem 1 in order to keep the exponential stability of the unforced system (1) during the process of stabilization of (2). Such an exponential stability of (1) ensures that the transformation of coordinates which is used in the proof of Theorem 2 is a Bohl transformation and thus, preserves the stability from the original coordinates. Note at this point that, contrarily to the time- invariant case, not always well- posed transformations of coordinates of linear time- varying systems preserve stability[6]. In that way, firstly the stability is proved in terms of ultimate boundedness of the system in the new coordinates obtained from that transformation. The system expressed in the original coordinates is also stable as a result of the fact that the transformation of coordinates used is proved to be a Bohl one. Thus, if all the switchings in the dynamics take place between distinct stable configurations then the choice of interval lengths provided by Theorem 1 for the homogeneous system is proved to guarantee Lyapunov' s stability with ultimate boundedness for (2).

If unstable dynamics are also allowed during certain time intervals of finite length then the lengths during all or some stabilization time subintervals can be required to be increased with respect to those provided by Theorem 1 for the unforced case. The increase of the lengths of the stabilization time subintervals can also be required when some time intervals, whose eigenvalues are stable, are not stabilization time intervals (in the sense of Theorem 1) since their lengths do not fulfill the minimum requested stabilization thresholds.

Generally speaking, the increase of the lengths of the stabilization subintervals increase as the lengths of possible intervals outside the stabilization set increase. In particular, such lengths have to satisfy the minimum thresholds established by Theorem 1 in order to guarantee the exponential stability of the homogeneous system.

CONCLUSION

This study has dealt with the robust exponential stability of slowly time-varying linear systems whose eigenvalues of the dynamics are not necessarily suitable for all time. All the eigenvalues are assumed to be strictly stable during certain, in general, possibly disjoint and connected stabilization time subintervals which have a duration exceeding some positive minimum threshold. The choice of lengths of such time subintervals from an appropriate time- scheduling rule has been used as the stabilization key tool of the overall time-varying system.

In that way, the unforced system becomes globally exponentially stable. It may also become robustly globally exponentially stable with the same set of stabilization time subintervals as in the unforced case under state-dependent disturbances. The robust exponential stability is achieved if the disturbances are sufficiently small and furthermore, their norm grows at most linearly, with sufficiently small slope, with the state norm. In the presence of a class of nonlinear state-dependent perturbation and / or vanishing disturbances, the system has proven to be locally exponentially stable around the equilibrium.

For bounded larger deviations of the initial conditions from the equilibrium, the system is still globally Lyapunov' s stable with ultimate boundedness provided that the possible switches in the system dynamics either end infinite time or, after some finite time, all switches (if any) occur towards configurations involving higher stability degrees than each current dynamics.

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